

# Registers

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# *Register*

- A *register* has two operations: *read()* and *write()*
- Sequential specification
  - *read()*
    - return(x)
  - *write(v)*
    - $x \leftarrow v$ ; return(ok)

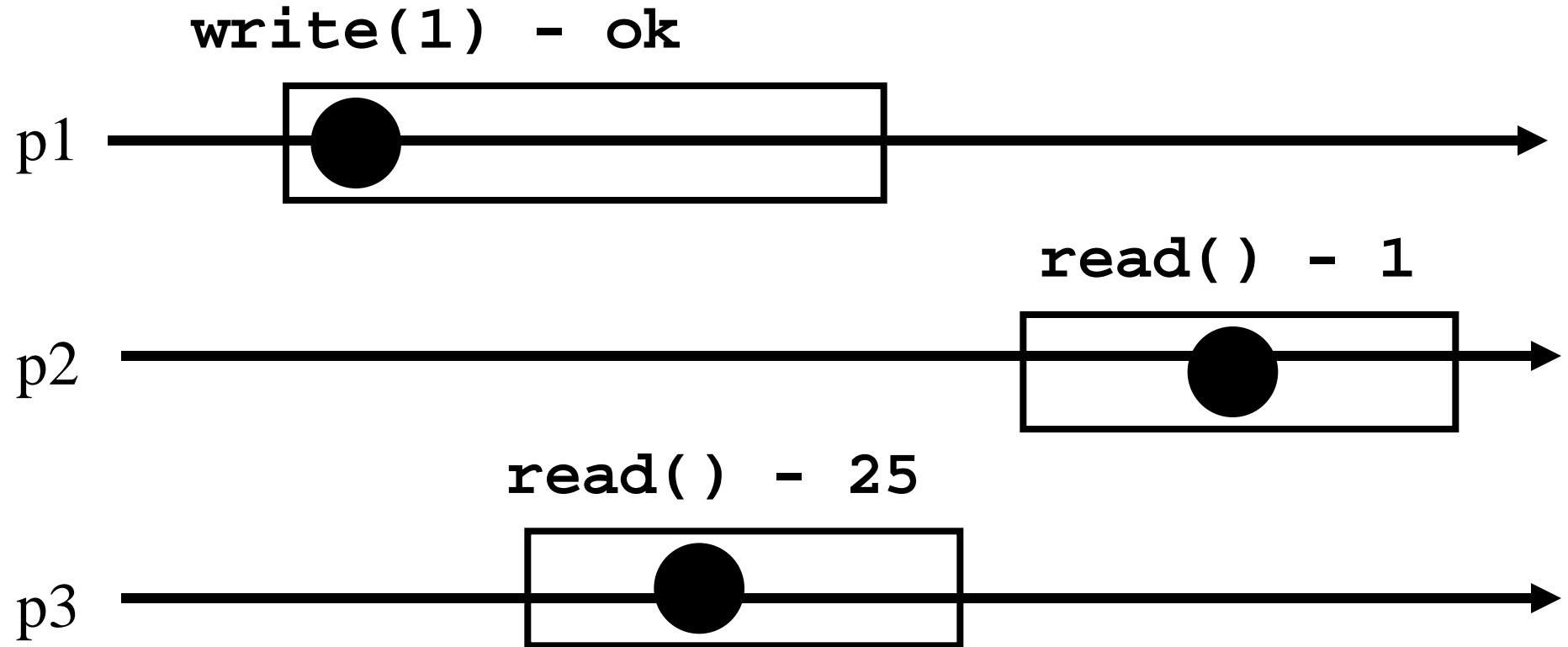
# Simplifications

- ☛ We assume that *registers* contain only integers
- ☛ Unless explicitly stated otherwise, *registers* are initially supposed to contain 0

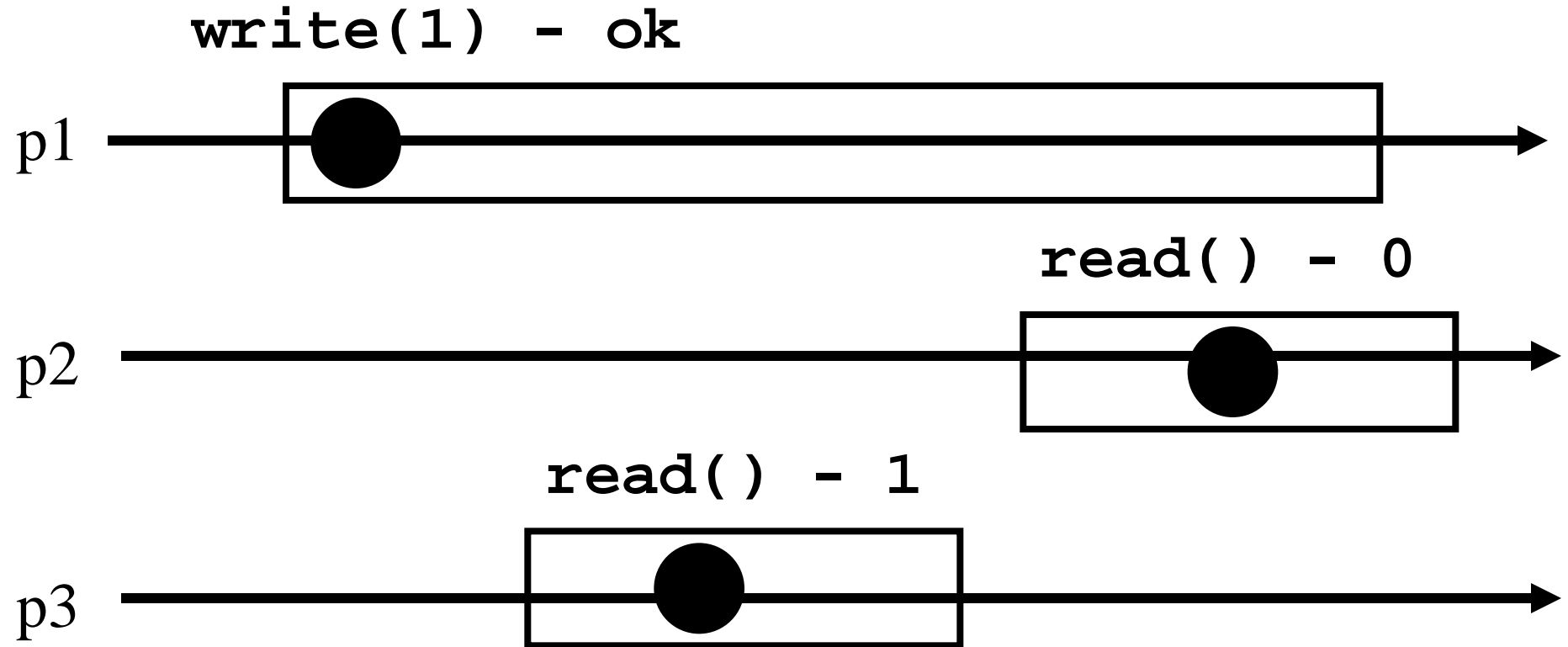
# Space of registers

- ☛ Dimension 1: binary (boolean) – multivalued
- ☛ Dimension 2:
  - ☛ SRSW (single reader, single writer)
  - ☛ MRSW (multiple reader, single writer)
  - ☛ MRMW (multiple reader, multiple writer)
- ☛ Dimension 3: safe – regular – atomic

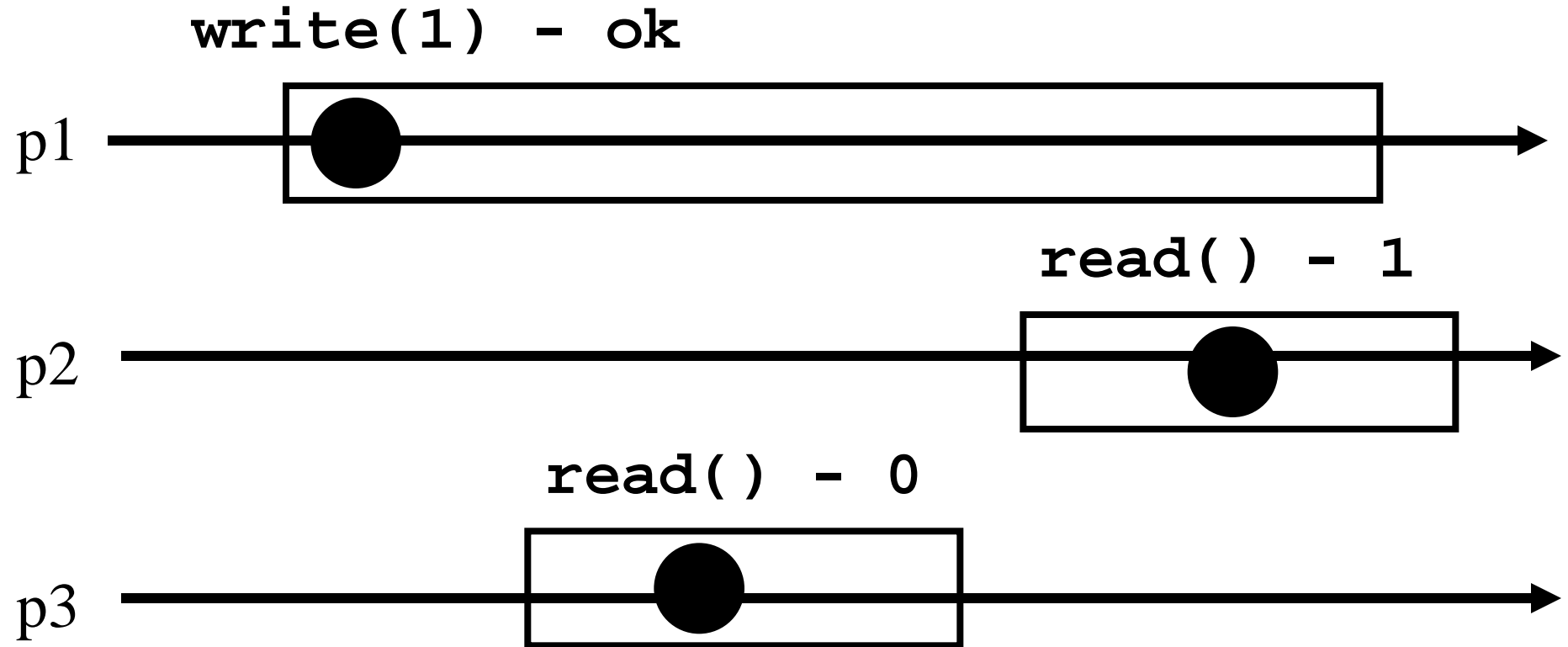
# Safe execution



# Regular execution



# Atomic execution



# 2 decades of hard work

- ☛ Theorem: A multivalued MRMW atomic *register* can be implemented with binary SRSW safe *register*



# Algorithms

- ☛ The process executing the code is implicitly assumed to be  $p_i$
- ☛ We assume a system of  $N$  processes
- ☛ NB. We distinguish base and high-level registers

# Conventions

- The operations to be implemented are denoted *Read()* and *Write()*
- Those of the base registers are denoted *read()* and *write()*
- We omit the *return(ok)* instruction at the end of *Write()* implementations

# From (binary) SRSW safe to (binary) MRSW safe

- ☛ We use an array of SRSW *registers*

Reg[1,...,N]

- ☛ **Read()**

- ☛ return (Reg[i].read());

- ☛ **Write(v)**

- ☛ for j = 1 to N

- ☛ Reg[j].write(v);

# From (binary) SRSW safe to (binary) MRSW safe

- ☛ The transformation works also for multi-valued registers and regular ones
- ☛ It does not however work for atomic registers

# From Binary MRSW safe to Binary MRSW regular

- We use one MRSW safe register

- **Read()**

- return(Reg.read());

- **Write(v)**

- if old  $\neq$  v then

- Reg.write(v);

- old := v;

# From Binary MRSW safe to Binary MRSW regular

- ☛ The transformation works for single reader *registers*
- ☛ It does not work for multi-valued *registers*
- ☛ It does not work for atomic *registers*

# From *binary* to *M-Valued*

## MRSW regular

- ☛ We use an array of MRSW registers  
Reg[0,1,...,M] init to [1,0,...,0]
- ☛ **Read()**
  - ☛ for  $j = 0$  to  $M$ 
    - ☛ if Reg[j].read() = 1 then return(j)
- ☛ **Write(v)**
  - ☛ Reg[v].write(1);
  - ☛ for  $j=v-1$  downto 0
    - ☛ Reg[j].write(0);

# From *binary* to *M-Valued* MRSW regular

- ☛ The transformation would not work if the Write() would first write 0s and then 1
- ☛ The transformation works for *regular* but *NOT* for *atomic* registers



# From SRSW *regular* to SRSW *atomic*

- ☛ We use one SRSW register Reg and two local variables t and x

## ☛ **Read()**

- ☛  $(t', x') = \text{Reg.read}();$
- ☛ if  $t' > t$  then  $t := t'; x := x';$
- ☛ return(x)

## ☛ **Write(v)**

- ☛  $t := t + 1;$
- ☛  $\text{Reg.write}(v, t);$

# From SRSW regular to SRSW atomic

- ☛ The transformation would not work for multiple readers
- ☛ The transformation would not work without timestamps  
(variable  $t$  representing logical time)

# From SRSW atomic to MRSW atomic

- We use  $N \times N$  SRSW atomic registers  $RReg[(1,1),(1,2),\dots,(k,j),\dots,(N,N)]$  to communicate among the readers
  - In  $RReg[(k,j)]$  the reader is  $p_k$  and the writer is  $p_j$
- We also use  $n$  SRSW atomic *registers*  $WReg[1,\dots,N]$  to store new values
  - the writer in all these is  $p_1$
  - the reader in  $WReg[k]$  is  $p_k$

# From SRSW atomic to MRSW atomic (cont'd)

## • **Write(v)**

- $t1 := t1 + 1;$
- for  $j = 1$  to  $N$ 
  - $WReg.write(v, t1);$

# From SRSW atomic to MRSW atomic (cont'd)

## Read()

- for  $j = 1$  to  $N$  do
  - $(t[j], x[j]) = \text{RReg}[i, j].\text{read}();$
- $(t[0], x[0]) = \text{WReg}[i].\text{read}();$
- $(t, x) := \text{highest}(t[..], x[..]);$
- for  $j = 1$  to  $N$  do
  - $\text{RReg}[j, i].\text{write}(t, x);$
- return( $x$ )

Value with highest  
timestamp

# From SRSW atomic to MRSW atomic (cont'd)

- ☛ The transformation would not work for multiple writers
- ☛ The transformation would not work if the readers do not communicate (i.e., if a reader does not write)

# From *MRSW* atomic to *MRMW* atomic

- We use  $N$  *MRSW* atomic registers  $\text{Reg}[1, \dots, N]$ ; the writer of  $\text{Reg}[j]$  is  $p_j$

## • **Write( $v$ )**

- for  $j = 1$  to  $N$  do
  - $(t[j], x[j]) = \text{Reg}[j].\text{read}();$
- $(t, x) := \text{highest}(t[..], x[..]);$
- $t := t + 1;$
- $\text{Reg}[i].\text{write}(t, v);$

# From MRSW atomic to MRMW atomic (cont'd)

## ☛ Read()

- ☛ for  $j = 1$  to  $N$  do
  - ☛  $(t[j], x[j]) = \text{Reg}[j].\text{read}();$
- ☛  $(t, x) := \text{highest}(t[..], x[..]);$
- ☛ return( $x$ )