Adopt-commit
Universal Constructions
BG-simulation

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- It offers the operation PROPOSE(*value*),

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Agreement	If a process returns $\langle commit, v \rangle$, the only pairs that can be returned are $\langle commit, v \rangle$ and $\langle adopt, v \rangle$.
Termination	An invocation of PROPOSE() by a correct process terminates.

Adopt-commit Implementation

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- All registers initialized to the special value $\perp$
- Two local sets a_i and b_i

Adopt-commit Implementation

```
 1: operation AC.PROPOSE(v_i)
 2:     A[i] ← v_i; a_i ← ∅
 3:     for j from 0 to n − 1 do
 4:         tmp_i ← A[j]
 5:         if tmp_i ≠ ⊥ then a_i ← a_i ∪ {tmp_i} end if
 6:     end for
 7:     if a_i = {v} then B[i] ← ⟨one, v⟩ else B[i] ← ⟨more, v_i⟩ end if
 8:     b_i ← ∅
 9:     for j from 0 to n − 1 do
10:         tmp_i ← B[j]
11:         if tmp_i ≠ ⊥ then b_i ← b_i ∪ {tmp_i} end if
12:     end for
13:     if b_i = {⟨one, v⟩} then
14:         return(⟨commit, v⟩)
15:     else if ∃⟨one, v⟩ ∈ b_i then
16:         return(⟨adopt, v⟩)
17:     else
18:         return(⟨adopt, v_i⟩)
19:     end if
20: end operation
```

- Validity and termination are straightforward.

Elements of Proof

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- Agreement: at most one value can appear with the tag *one*.

- Implements consensus.

Adopt-commit-based Consensus

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- Implementation stripped in sequence of asynchronous rounds.
- Based on an infinite array of Adopt-commit objects $AC[r]$, $r \geq 0$.
- A shared MWMR register DEC initialized to $\perp$.

```
1:  operation CONS.PROPOSE(v_i)
2:      est_i ← v_i; r_i ← 0
3:      while DEC = ⊥ do
4:          if leader_i = i then
5:              r_i ← r_i + 1
6:              ⟨tag_i, val_i⟩ ← AC[r_i].PROPOSE(est_i)
7:              if tag_i = commit then
8:                  DEC ← val_i
9:              else
10:                 est_i ← val_i
11:             end if
12:         end if
13:     end while
14:     return DEC
15: end operation
```

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- If eventually one and only one correct process verifies $leader_i = i$ then any correct process eventually decides.

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Termination
- If eventually one and only one correct process verifies $leader_i = i$ then any correct process eventually decides.
- If all processes verify $leader_i = i$ forever, the algorithm is only *obstruction-free*.

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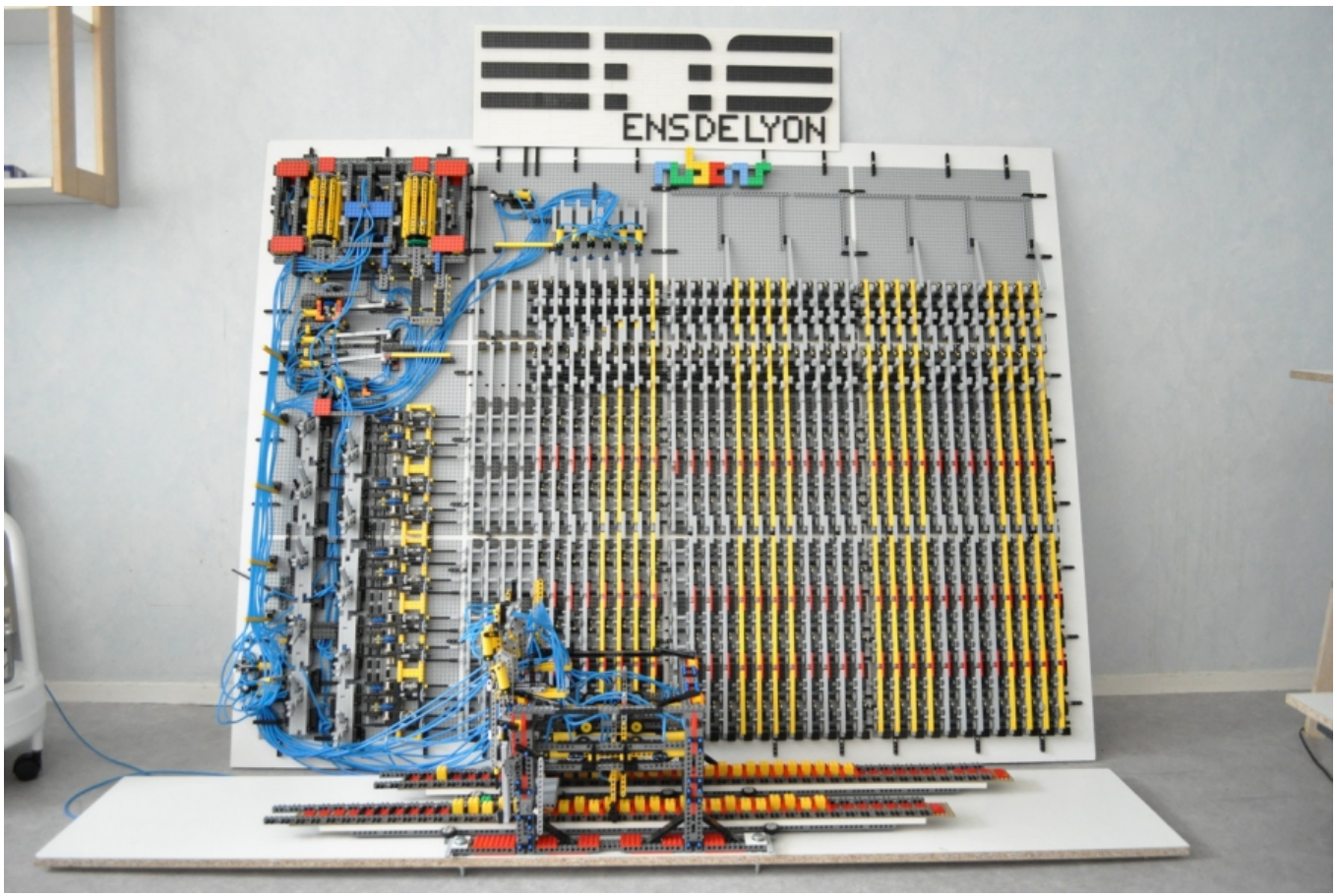
Universality of the Turing Machine



Consensus is *universal*

Any object O following a sequential specification can be implemented, in a wait-free and linearizable manner, from atomic registers and consensus objects.[1]

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Consensus is *universal*

Any object O following a sequential specification can be implemented, in a wait-free and linearizable manner, from atomic registers and consensus objects.[1]

If we know how to solve consensus in our system, we can implement a highly available Turing machine.

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What kind of universality can we achieve
without consensus?

A weaker agreement: k-set agreement.

Interface Offers a $\textsc{propose}(v)$ operation that returns a value.

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Interface Offers a PROPOSE(v) operation that returns a value.

Validity Decided values are proposed values.

Termination Any invocation of PROPOSE by a correct process terminates.

Agreement No more than k different values are decided in the system.

Another generalization of consensus: k-simultaneous consensus.

Interface Offers a $\textsc{propose}(v_1, \ldots, v_k)$ operation that returns a pair $(\mathit{index}, \mathit{value})$, $\mathit{index} \in \{1, \ldots, k\}$.

Another generalization of consensus: *k*-simultaneous consensus.

Interface Offers a PROPOSE($v_1, \ldots, v_k$) operation that returns a pair (*index*, *value*), *index* $\in \{1, \ldots, k\}$.

Validity If a PROPOSE operation returns (i, v), then a process invoked PROPOSE($v_1, \ldots, v_k$) with $v_i = v$.

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Validity If a PROPOSE operation returns (i, v), then a process invoked PROPOSE$(v_1, \ldots, v_k)$ with $v_i = v$.

Termination Any invocation of PROPOSE by a correct process terminates.

Agreement If two PROPOSE operations return (i, v) and (i', v') with $i = i'$, then $v = v'$.

- Both 1-set agreement and 1-simultaneous consensus are equivalent to consensus.

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- k-set agreement and k simultaneous consensus are equivalent in asynchronous shared memory systems in presence of an arbitrary number of crashes.
- k-set agreement cannot be implemented in asynchronous shared memory systems prone to $t \geq k$ crashes.

Generalized Universality

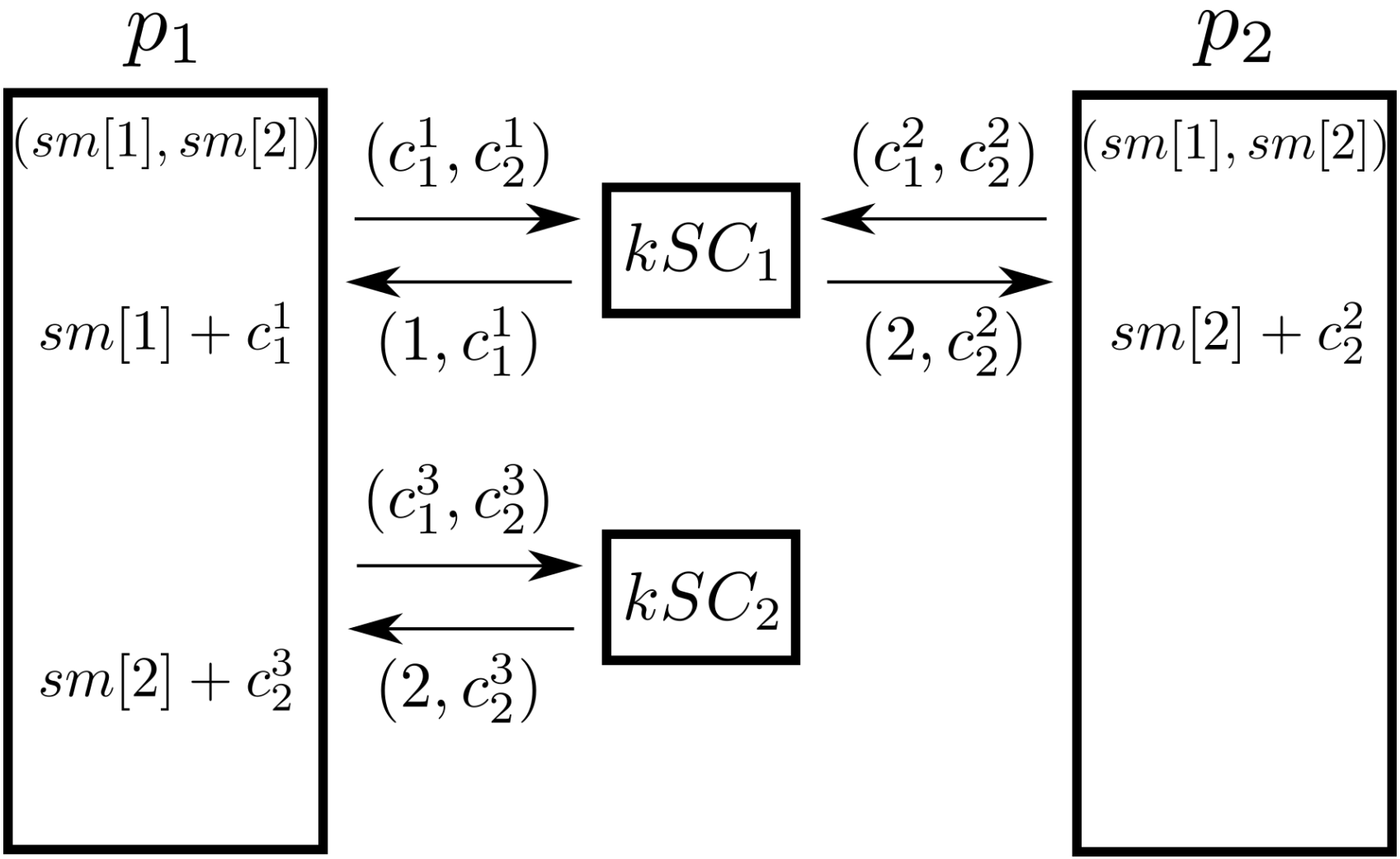
From k-simultaneous consensus objects and registers, it is possible to implement k shared objects of which at least one is highly available[2].

[2] Gafni E. and Guerraoui R., *Generalizing universality.* CONCUR (2011)

From Standard Universal Construction...

1: **while** true **do**
2: $c \leftarrow commands.next()$
3: $CONS \leftarrow consensus.next()$
4: $c' \leftarrow CONS.\text{PROPOSE}(c)$
5: $sm.perform(c')$
6: **end while**

1: **while** true **do**
2: **for** j from 1 to k **do**
3: $c[j] \leftarrow commands[j].next()$
4: **end for**
5: $kSC \leftarrow k\text{-}sim\text{-}cons.next()$
6: $(i, dc) \leftarrow kSC.\text{PROPOSE}(c[1], \ldots, c[k])$
7: $sm[i].perform(dc)$
8: **end while**


p_1
p_2
$(sm[1], sm[2])$
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(c_1^1, c_2^1)
(c_1^2, c_2^2)
kSC_1
$sm[1] + c_1^1$
$(1, c_1^1)$
$(2, c_2^2)$
$sm[2] + c_2^2$
(c_1^3, c_2^3)
kSC_2
$sm[2] + c_2^3$
$(2, c_2^3)$

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- They need to communicate the commands they apply and to retrieve the commands of the other processes.
- Adopt-commit objects may help...

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 1: while true do
 2:     for j from 1 to k do
 3:         if c[j] = ⊥ then c[j] ← commands[j].next() end if
 4:     end for
 5:     kSC ← k-sim-cons.next()
 6:     (i, dc) ← kSC.PROPOSE(c[1], ..., c[k])
 7:     for j from 1 to k do
 8:         AC[j] ← adopt-commit[j].next()
 9:         if i = j then
10:             ⟨tag[j], ac_com[j]⟩ ← AC[j].PROPOSE(dc)
11:         else
12:             ⟨tag[j], ac_com[j]⟩ ← AC[j].PROPOSE(c[j])
13:         end if
14:         if tag[j] = commit then
15:             sm[j].perform(ac_com[j]); c[j] ← ⊥
16:         else
17:             c[j] ← ac_com[j]
18:         end if
19:     end for
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p_1 $(1, c_1^1) \leftarrow kSC.\text{PROPOSE}(c_1^1, c_2^1)$

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Problem with Liveness

$$
\begin{aligned}
p_1 \quad & (1, c_1^1) \leftarrow kSC.\textsc{propose}(c_1^1, c_2^1) \\
p_2 \quad & (2, c_2^2) \leftarrow kSC.\textsc{propose}(c_1^2, c_2^2) \\
p_1 || p_2 \quad & AC[1].\textsc{propose}(c_1^1) || AC[1].\textsc{propose}(c_1^2)
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\end{array}
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The four adopt-commit can return $\langle adopt, - \rangle$...
It can be repeated forever without any progress.

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Exploit Success First

Let's launch the processes first on the machines returned by the k-simultaneous consensus.

Exploit Success First

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 7:     AC[i] ← adopt-commit[i].next()
 8:     ⟨tag[i], ac_com[i]⟩ ← AC[i].PROPOSE(dc)
 9:     for j from 1 to k, j ≠ i do
10:         AC[j] ← adopt-commit[j].next()
11:         ⟨tag[j], ac_com[j]⟩ ← AC[j].PROPOSE(c[j])
12:         if tag[j] = commit then
13:             sm[j].perform(ac_com[j]); c[j] ← ⊥
14:         else
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- and (b) already finished executing $AC[i'].\text{PROPOSE}$.

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- and (b) already finished executing $AC[i'].\text{PROPOSE}$.
- If p_y didn't commit, then another process concurrently accessed $AC[i']$.
- But it cannot be p_x!
- Now there is at least a commit per round.

Still some Safety Problems

We now commit at each round and each process at least adopts each of the committed values. But commands can be skipped.

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p_1 commits c' for machine m

p_2 commits/adopt c' for m?

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p_2 commits c for machine m

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- Piggy-backing the previous command in the currently proposed one.

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- Histories can also be exchanged directly through the shared memory.[3]

[3] Michel Raynal, Julien Stainer, Gadi Taubenfeld: *Distributed Universality*. OPODIS (2014)

From k-simultaneous consensus objects and atomic register, it is possible to simulate k state-machines such that at least one always progresses.

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- It can become wait-free by the use of helping:
  - Processes write in shared memory the commands they plan to execute on the machines.
  - While deciding the next command to apply on a machine m, processes check the number of commands nc that have been applied to m.
  - If process p_x with $x = nc \mod n$ has written a command that has not been executed, then other processes propose it as $(nc + 1)$-th command to execute on m.

- When there is no contention (e.g. a process is far ahead), the use of the k-simultaneous consensus object can be avoided, at the cost of more adopt-commit objects.

- When there is no contention (e.g. a process is far ahead), the use of the k-simultaneous consensus object can be avoided, at the cost of more adopt-commit objects.
- To guarantee that several machines progress, the k-simultaneous consensus objects can be replaced by more powerful objects.

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In a crash-free system, safe-agreement objects implement consensus.

1: **init** $REG[0, \ldots, n-1] \leftarrow [\langle \perp, 0 \rangle]$
2: **operation** PROPOSE(v)
3: $REG[i] \leftarrow \langle v, 1 \rangle$
4: $snap_i \leftarrow REG.snapshot()$
5: **if** $\exists x \ : \ snap_i[x].level = 2$ **then**
6: $REG[i] \leftarrow \langle v, 0 \rangle$
7: **else**
8: $REG[i] \leftarrow \langle v, 2 \rangle$
9: **end if**
10: **end operation**
11: **operation** DECIDE( )
12: **repeat**
13: $snap_i \leftarrow REG.snapshot()$
14: **until** $\forall x \ : \ snap_i[x].level \neq 1$
15: $x \leftarrow \min \{y \mid snap_i[y] = 2\}$
16: return $snap_i[x].value$
17: **end operation**

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- $t+1$ simulators $q_0, \ldots, q_t$ among which up to t may crash.
- n simulated processes $p_0, \ldots, p_{n-1}$ communicating by writes and snapshots.

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To preserve coherence, simulators have to agree on the snapshots taken by the simulated processes.

1: **init** $r_i \leftarrow 1$
2: **while** p_i not decided **do**
3: simulate its r_i-th write on behalf of p_i
4: simulate its r_i-th snapshot on behalf of p_i
5: propose this snapshot to the r_i-th safe-agreement object associated to p_i
6: decide on a snapshot from this safe-agreement object
7: compute the new state of p_i
8: $r_i \leftarrow r_i + 1$
9: **end while**

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- But the crash of a simulator can block more than one simulated process.

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2: **while** p_i not decided **do**
3: simulate its r_i-th write on behalf of p_i
4: simulate its r_i-th snapshot on behalf of p_i
5: enter mutex
6: propose this snapshot to the r_i-th safe-agreement object associated to p_i
7: exit mutex
8: decide on a snapshot from this safe-agreement object
9: compute the new state of p_i
10: $r_i \leftarrow r_i + 1$
11: **end while**

- Thanks to the mutex, a simulator never participates to more than one safe-agreement propose operation.

- Thanks to the mutex, a simulator never participates to more than one safe-agreement propose operation.
- The crash of a simulator consequently blocks at most one simulated process.

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- k-set agreement is impossible in a system of $k+1$ processes with k crashes
  $\Longrightarrow$ k-set agreement is impossible in a system of 100 processes with k crashes.

What matters in a system is not the number of processes but the maximum number of crashes.

- Adopt-commit and adopt-commit-based consensus
- Universal construction from k-simultaneous consensus objects and registers
- BG-simulation