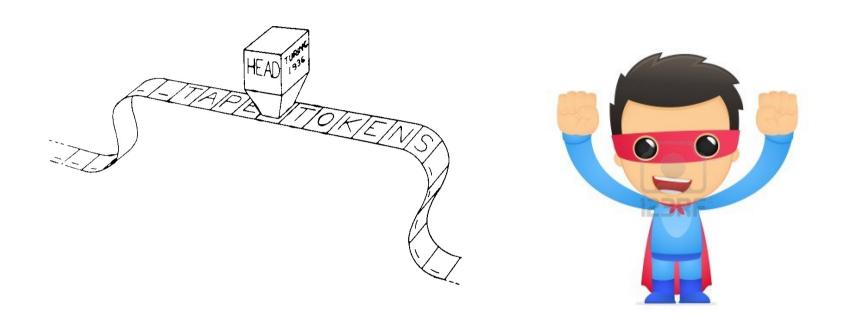
Classical Universality: Turing

Universality of Consensus [Lamport-Schneider-Herlihy-CT]



Linearizable (atomic)

Highly-available (wait-free)

Act1: Universality Turing

Act 2: Modern Universality
Turing + Lamport

Consensus

Processes propose each a value and agree on one

$$\begin{array}{c|c}
 & c \\
\hline
 & c
\end{array} \qquad consensus \qquad \begin{array}{c}
 & c' \\
\hline
 & c
\end{array} \qquad p2$$

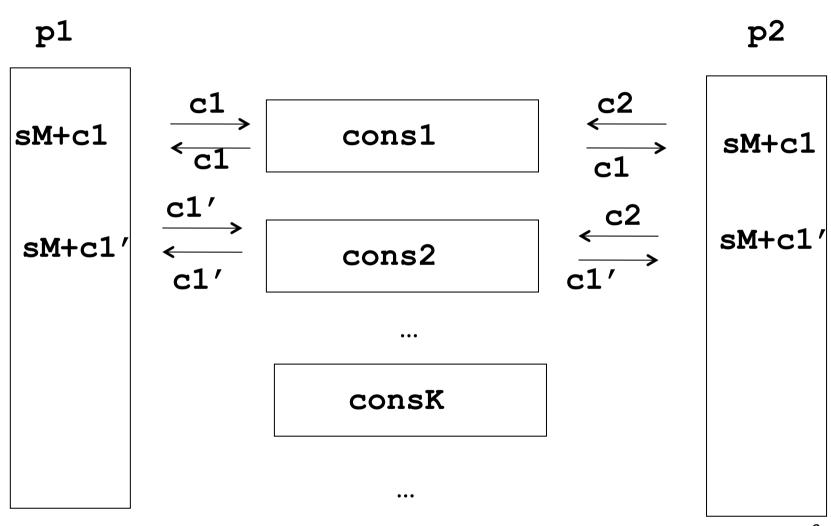
Each process holds a copy of the simulated machine

Each process holds a list of commands for the machine

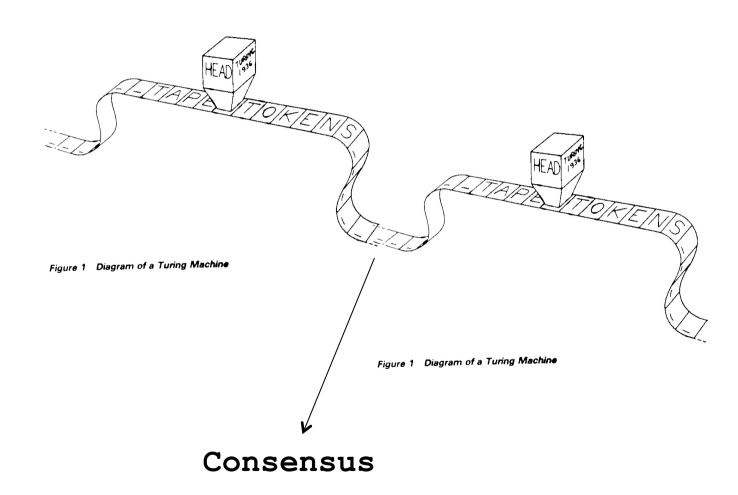
All processes share a list of consensus objects

p1 p2 cons1 sM sM cons2 consK

- while(true)
- c = commands.next()
- cons = Consensus.next()
- c' = cons.propose(c)
- sM.perform(c')



What if consensus is not ensured?



K-Consensus

Every process proposes a vector of k values and returns a value at some position (Chauduri et al)

$$(i,c) = propose(kVect)$$

$$vect$$

$$k-consensus$$

$$(i,c)$$

$$vect$$

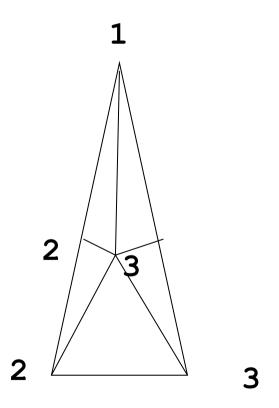
$$(i',c')$$

p1

K-Consensus

- Validity: the value returned at any position has been proposed at that position
- Agreement: no two values returned at the same position are different
- Termination: every correct process that proposes eventually returns

k+1-consensus is strictly weaker than k-consensus in any system of more than k+1 processes (Godel prize 2004 – HS,BG,SZ 93)



Sperner's Lemma: at least one triangle has three colors

What form of universality with K-consensus?

With consensus

We implement a highly-available state machine



With k-consensus

We implement k state machines of which *at least* one is highly-available

Generalized Universality

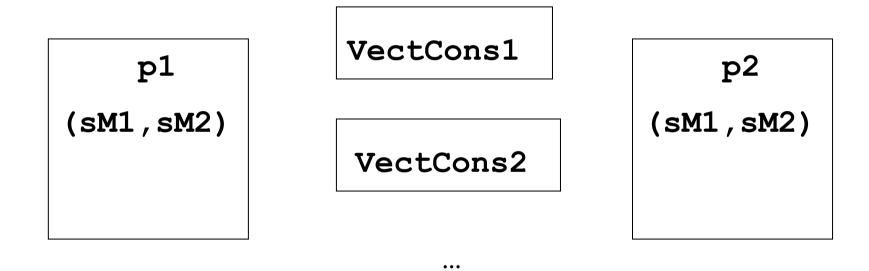
Act1: Universality Turing

Act 2: Modern Universality
Turing + Lamport

Act 3: Generalized Universality

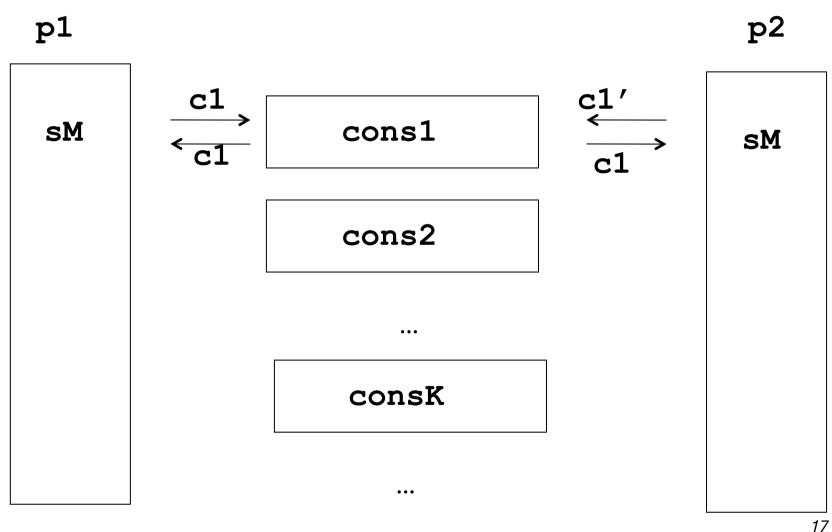
Generalized Universality

Each process holds a copy of each of the machines sM(i) - and a lists of commands for each



The processes share a list of k-vector consensus objects

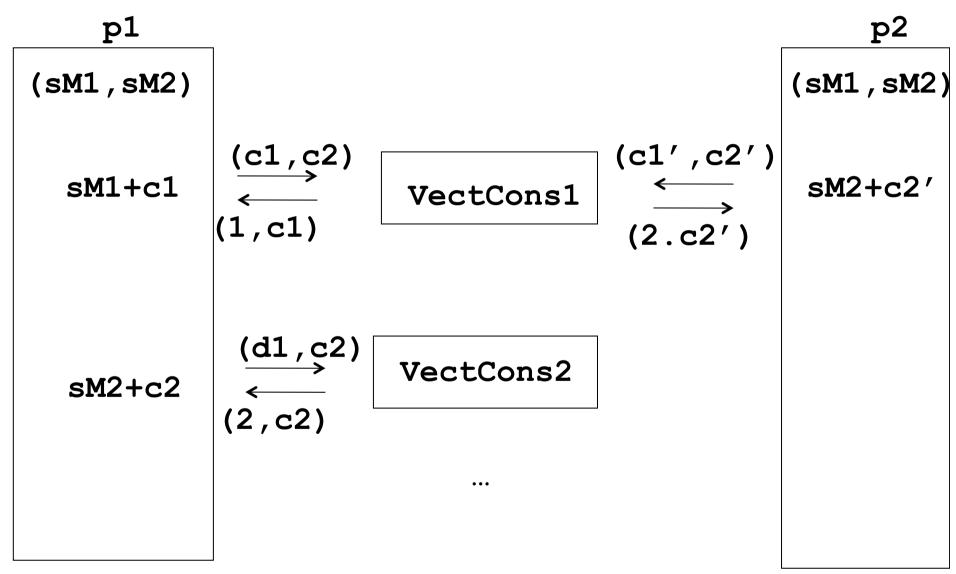
- while(true)
- c = commands.next()
- cons = consensus.next()
- c' = cons.propose(c)
- sM.perform(c')



Generalized Universality? Problem with safety

- while(true)
- for j = 1 to k: com(j) = commands(j).next()
- kVectC = kVectCons.next()
- (c,i) = kVectC.propose(com)
- sM(i).perform(c)

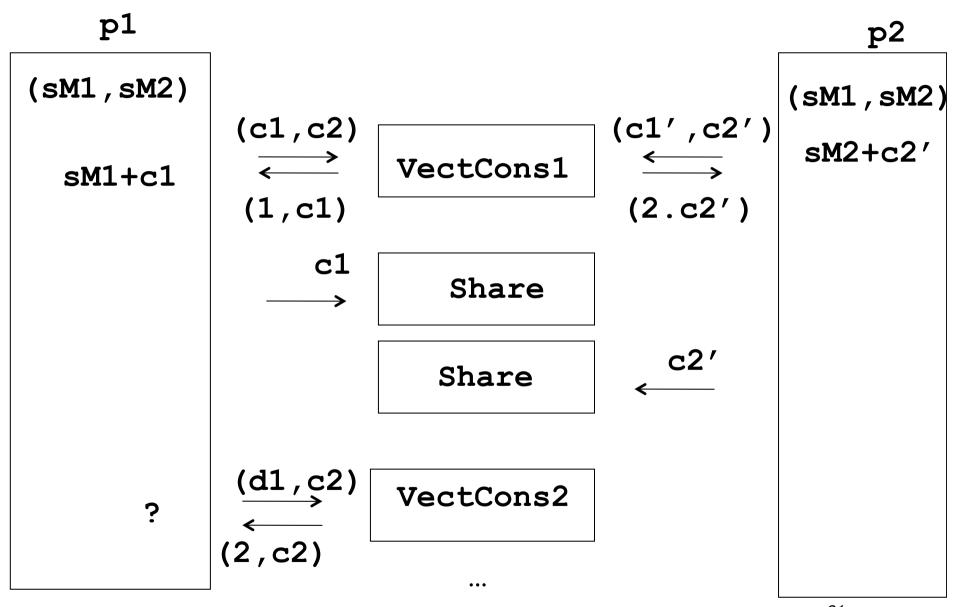
Problem with safety



Generalized Universality

- while(true)
- for j = 1 to k: com(j) = commands(j).next()
- kVectC = kVectCons.next()
- (c,i) = kVectC.propose(com)
- check other processes for any missing c'
- sM(i).perform(c)
- inform other processes about c

Generalized Universality



1st key idea (ensuring safety)

$$\begin{array}{c}
c1 & c2 \\
 \longrightarrow \\
 \text{commit(c)} & \text{adopt/commit} & \longrightarrow \\
 & \text{adopt(c)}
\end{array}$$

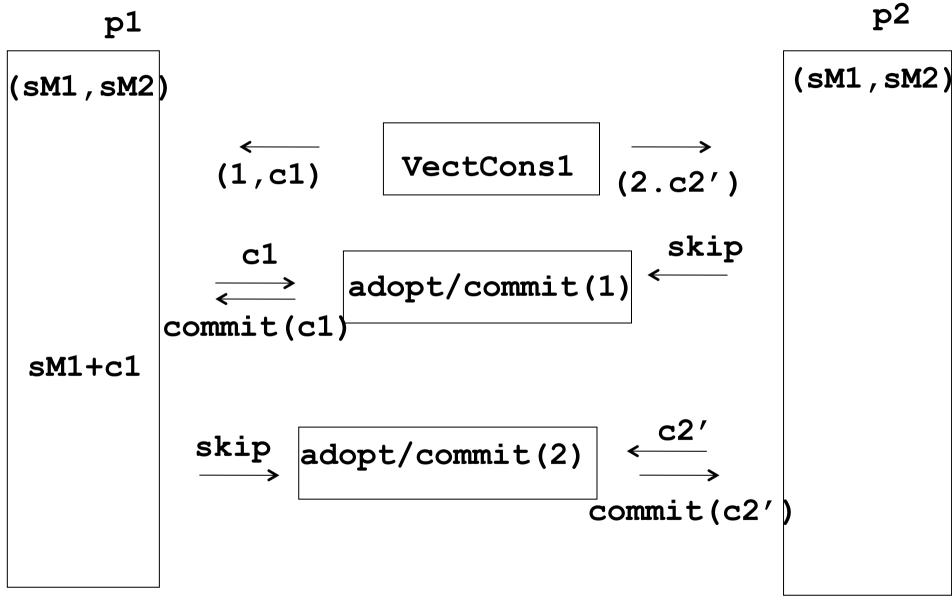
write (c)
if there is only c, write (commit, c)
if there is only (commit, c), return(commit, c)
if there is (commit, c'), return(adopt, c')
else return (adopt, c)

Adopt/commit

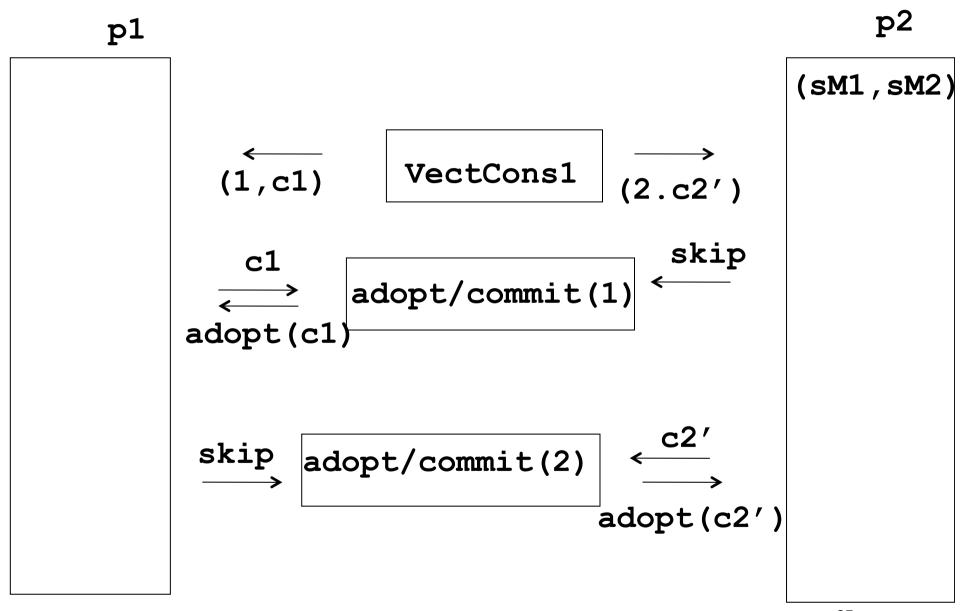
Invariant (1): if a value v is committed then no other value is returned

Invariant (2): if all processes propose the same command then the command is committed

Generalized Universality



Problem with liveness



2nd key idea (ensuring liveness) Exploit success first

Can it be that no command is committed? i.e., if every adopt/commit box has one process proposes skip

Generalized universality (step 0)

- newCom = commands.next()
- while(true)
- kVectC = kVectCons.next()

Generalized universality (step 1)

- . . .

(c,i) = kVectC.propose(newCom)

•

Generalized universality (step1-2)

. . . .

- (c,i) = kVectC.propose(newCom)
- vect(i) = commitment(i,c)

-

Generalized universality (step1-2-2')

```
...
```

- (c,i) = kVectC.propose(newCom)
- vect(i) = commitment(i,c)
- for j = 1 to k except i:
 - vect(j) = commitment(newCom(j))

. . .

Generalized universality (step 3)

```
for i = 1 to k
    if ok(vect(i)) then
        sM(i).perform(vect(i))
        newCom(i) = commands(i).next()
    else
        newCom(i) = vect(i)
```

Generalized universality (step 3')

```
for i = 1 to k
If older(newCom(i),vect(i)) then
                        sM(i).perform(newCom(i))
If no(vect(i)) then newCom(i) = vect(i)
else
sM(i).perform(vect(i))
If vect(i) = newCom(i) then
  newCom(i) = commands(i).next()
add(newCom(i),vect(i))
```

Commitment

 Safety: a process does not perform a command unless all others know the command

 Liveness: at least one process executes a command in every round

NB. Every correct process executes at least one command every two rounds

Act1: Universality Turing

Act 2: Modern Universality
Turing + Lamport

Act 3: Generalized Universality