

## Exercise 8

**Problem 1.**

Devise an obstruction-free, anonymous algorithm that implements binary consensus using a **finite** number of (unbounded) counters.

**Reminder:** a counter object implements two operations: *inc*, which increments the value of the counter and returns *ok*, and *read*, which returns the current value of the counter.

**Solution**

The following algorithm solves the problem:

**uses:**  $C_0, C_1$  – counters

**upon** *propose*( $v$ ) **do**

**while** *true* **do**

$(x_0, x_1) \leftarrow \text{readCounters}();$

**if**  $x_0 > x_1$  **then**  $v \leftarrow 0;$

**else if**  $x_1 > x_0$  **then**  $v \leftarrow 1;$

**if**  $|x_0 - x_1| \geq n$  **then return**  $v;$

$C_v.\text{inc}();$

The *readCounters* procedure atomically reads both counters  $C_0$  and  $C_1$ . It can be implemented as follows:

**upon** *readCounters*() **do**

**while** *true* **do**

$x_0 \leftarrow C_0.\text{read}();$

$x_1 \leftarrow C_1.\text{read}();$

$x'_0 \leftarrow C_0.\text{read}();$

**if**  $x_0 = x'_0$  **then return**  $(x_0, x_1);$

**Problem 2.** According to the implementation of anonymous snapshot given in the lecture, if we place

“If some  $\text{Reg}[j]$  contains a collect with a higher timestamp than  $ts$ , then return that collect”

by the following

“If some  $\text{Reg}[j]$  contains a collect with a timestamp *no less than*  $ts$ , then return that collect”,

is the implementation still correct? Justify your answer.

## Solution

The answer is yes. To justify this, we show linearizability and termination still hold. For linearizability, we need only to justify the return value of the replaced condition. Consider the first scan  $s$  which returns on this condition. (The “first” scan refers to when the scan starts.) Since the timestamp  $\tau$  of the snapshot  $ret$  returned by  $s$  is no less than  $ts$  (which is obtained at the beginning of  $s$ ), therefore the  $wInc$  procedure which returns  $\tau$  (denoted by  $wInc_1$ ) cannot end before the  $wInc$  procedure which returns  $ts$  (denoted by  $wInc_2$ ) starts, by the property of the weak counter. In other words,  $wInc_1$  ends no earlier than  $wInc_2$  starts. Thus the call of scan (denoted by  $s_{ret}$ ) inside the update which writes  $ret$  ends no earlier than  $s$  starts. I.e., two scans  $s$  and  $s_{ret}$  are concurrent. As a result,  $s$  can be linearized at the same point as  $s_{ret}$ . Since  $s_{ret}$  returns a linearizable value, then  $s$  also returns a linearizable value. We can extend the reasoning to infinity by induction. For termination, it is easy to see that now the implementation has more chances to return, and therefore must satisfy termination (as the original implementation satisfies termination).