

# Computing with anonymous processes

*Prof R. Guerraoui  
Distributed Computing Laboratory*



# Counter (sequential spec)

- ➊ A **counter** has two operations *inc()* and *read()* and maintains an integer  $x$  init to 0
- ➋ **read():**
  - ➌ return( $x$ )
- ➌ **inc():**
  - ➍  $x := x + 1;$
  - ➎ return(ok)

# Counter (atomic implementation)

- ☞ The processes share an array of SWMR registers Reg[1,...,n] ; the writer of register Reg[i] is pi
- ☞ ***inc():***
  - ☞ temp := Reg[i].read() + 1;
  - ☞ Reg[i].write(temp);
  - ☞ return(ok)

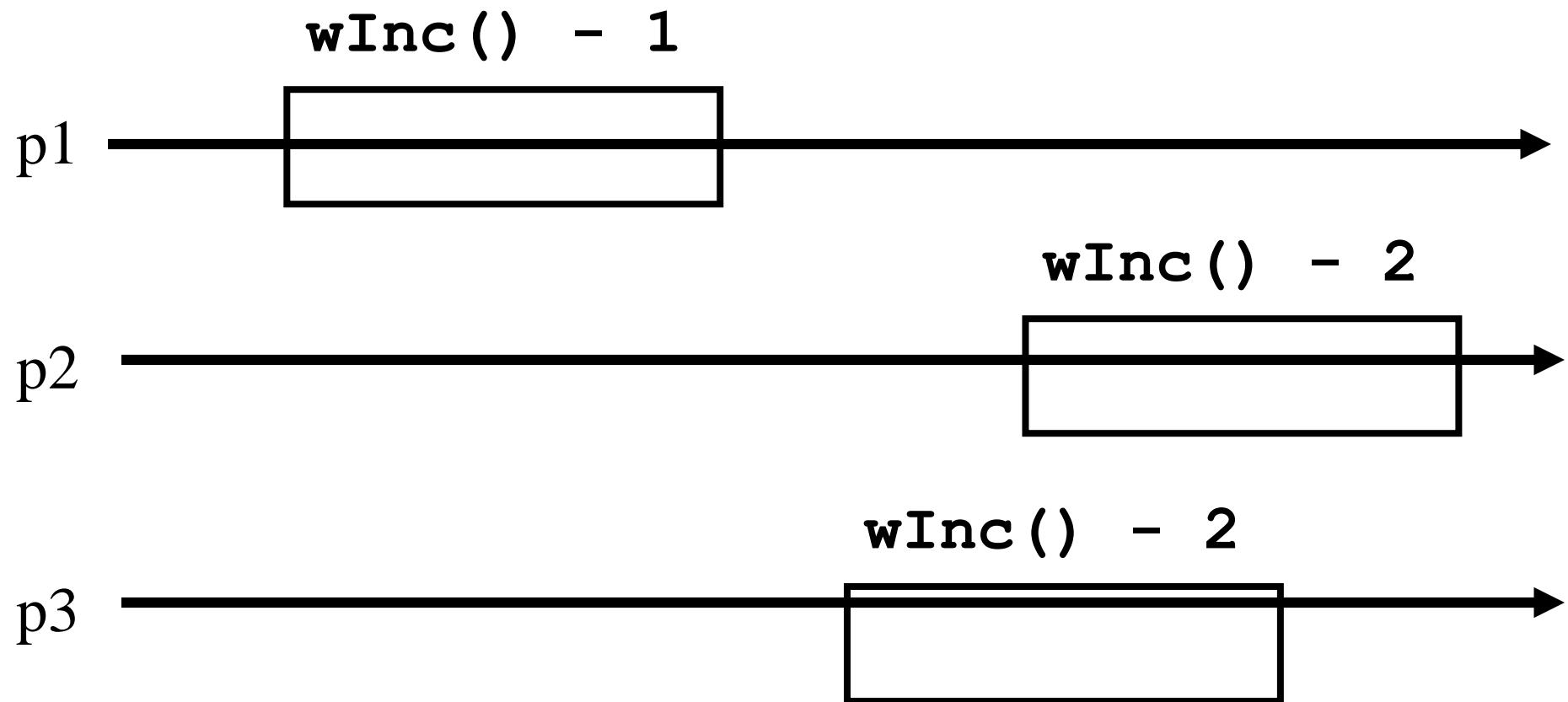
# Counter (atomic implementation)

```
➊ read():
 ➋   sum := 0;
 ➋   for j = 1 to n do
    ⌄     sum := sum + Reg[j].read();
 ⌄   return(sum)
```

# Weak Counter

- A ***weak counter*** has one operation ***wInc()***
- ***wInc()*:**
  - $x := x + 1;$
  - $\text{return}(x)$
- Correctness: if an operation precedes another, then the second returns a value that is larger than the first one

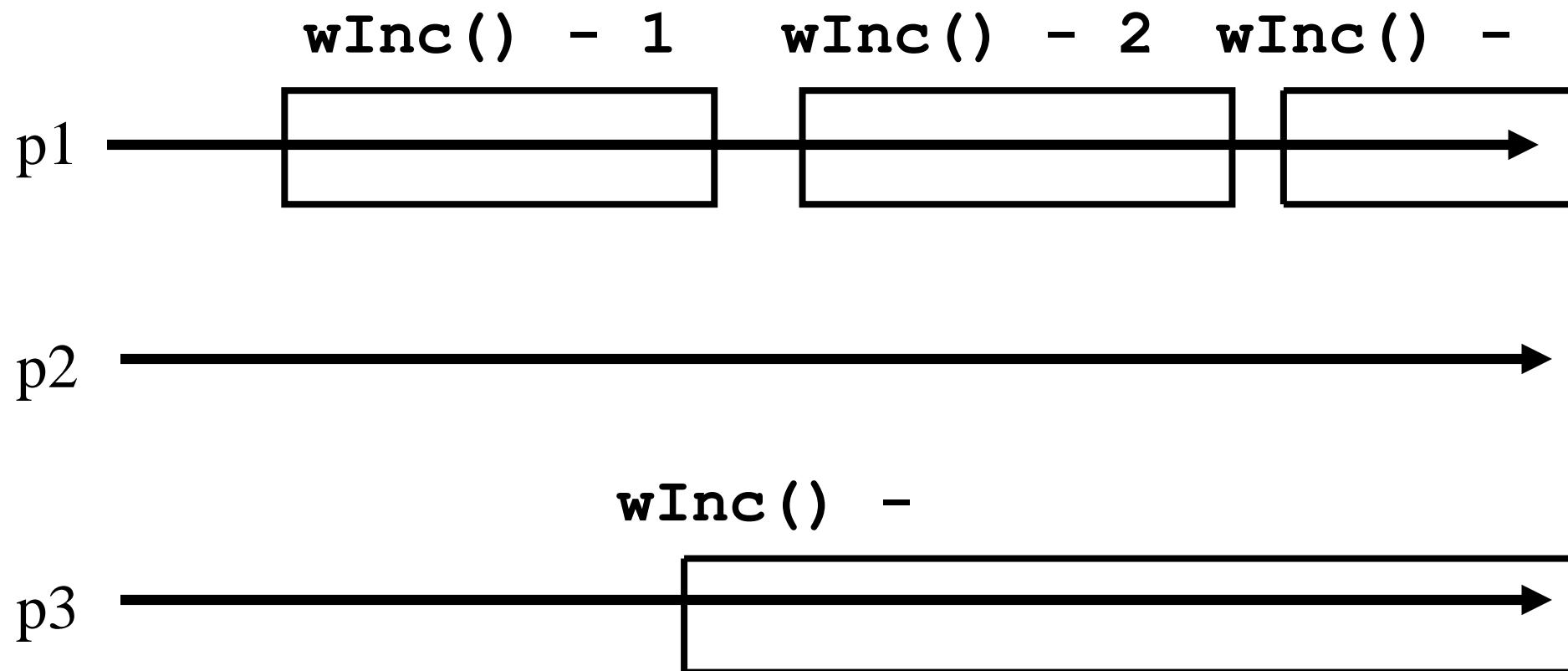
# Weak counter execution



# Weak Counter (lock-free implementation)

- ☞ The processes share an (infinite) array of MWMR registers  $\text{Reg}[1, \dots, n, \dots]$ , init to 0
- ☞ ***wInc()*:**
  - ☞  $i := 1;$
  - ☞ while ( $\text{Reg}[i].\text{read}() \neq 0$ ) do
    - ☞  $i := i + 1;$
    - ☞  $\text{Reg}[i].\text{write}(1);$
    - ☞ return( $i$ );

# Weak counter execution



# Weak Counter (wait-free implementation)

- ↙ The processes also use a MWMR register L
- ↙ **wInc():**
  - ↙ i := 1;
  - ↙ while (Reg[i].read() ≠ 0) do
    - ↙ if L has been updated n times then
      - ↙ return the largest value seen in L
    - ↙ i := i + 1;
  - ↙ L.write(i);
  - ↙ Reg[i].write(1);
  - ↙ return(i);

# Weak Counter (wait-free implementation)

## • *wInc()*:

- $t := l := L.read(); i := 1; k := 0;$
- $\text{while } (\text{Reg}[i].read() \neq 0) \text{ do}$ 
  - $i := i + 1;$
  - $\text{if } L.read() \neq l \text{ then}$ 
    - $l := L.read(); t := \max(t, l); k := k + 1;$
    - $\text{if } k = n \text{ then return}(t);$
  - $L.write(i);$
- $\text{Reg}[i].write(1);$
- $\text{return}(i);$

# Snapshot (sequential spec)

- ➊ A ***snapshot*** has operations ***update()*** and ***scan()*** and maintains an array  $x$  of size  $n$
- ➋ ***scan():***
  - ➌ return( $x$ )
- ➌ NB. No component is devoted to a process
- ➍ ***update( $i, v$ ):***
  - ➎  $x[i] := v;$
  - ➏ return(ok)

# Key idea for atomicity & wait-freedom

- ➊ The processes share a ***Weak Counter***.  
Wcounter, init to 0;
- ➋ The processes share an array of ***registers***  
Reg[1,...,N] that contains each:
  - ➌ a value,
  - ➌ a timestamp, and
  - ➌ a copy of the entire array of values

# Key idea for atomicity & wait-freedom (cont'd)

- To ***scan***, a process keeps collecting and returns a collect if it did not change, or some collect returned by a concurrent ***scan***
  - Timestamps are used to check if a scan has been taken in the meantime
- To ***update***, a process ***scans*** and writes the value, the new timestamp and the result of the scan

# Snapshot implementation

Every process keeps a local timestamp ts

☛ ***update(i,v):***

- ☛ ts := Wcounter.wInc();
- ☛ Reg[i].write(v,ts,self.scan());
- ☛ return(ok)

# Snapshot implementation

- ➊ ***scan():***

- ➋ ts := Wcounter.wInc();
  - ➋ while(true) do
    - ➋ If some Reg[j] contains a collect with a higher timestamp than ts, then return that collect
    - ➋ If n+1 sets of reads return identical results then return that one

# Consensus (obstruction-free)

- ☞ We consider binary consensus
- ☞ The processes share two infinite arrays of registers:  $\text{Reg}_0[i]$  and  $\text{Reg}_1[i]$
- ☞ Every process holds an integer  $i$  init to 1
- ☞ Idea: to impose a value  $v$ , a process needs to be fast enough to fill in registers  $\text{Reg}_v[i]$

# Consensus (obstruction-free)

## ☞ ***propose(v):***

```
    ☞ while(true) do
        ☞ if Reg1-v[i] = 0 then
            ☞     Regv[i] := 1;
            ☞     if i > 1 and Reg1-v[i-1] = 0 then
                ☞         return(v);
            ☞     else v:= 1-v;
            ☞     i := i+1;
    end
```

# Consensus (solo process)

$q(1)$

$\text{Reg0}(1) = 0$

$\text{Reg1}(1) := 1$

$\text{Reg0}(2) = 0$

$\text{Reg1}(2) := 1$

$\text{Reg0}(1) = 0$

# Consensus (lock-step)

$q(1)$

$\text{Reg0}(1) = 0$

$\text{Reg1}(1) := 1$

$\text{Reg0}(2) = 0$

$\text{Reg1}(2) := 1$

$\text{Reg0}(1) = 1$

$p(0)$

$\text{Reg1}(1) = 0$

$\text{Reg0}(1) := 1$

$\text{Reg1}(2) = 0$

$\text{Reg0}(2) := 1$

$\text{Reg0}(1) = 1$

# Consensus (binary)

- ➊ ***propose(v):***

- ➋ while(true) do
    - ⌂ If  $\text{Reg}_{1-v}[i] = 0$  then
      - ⌃  $\text{Reg}_v[i] := 1;$
      - ⌃ if  $i > 1$  and  $\text{Reg}_{1-v}[i-1] = 0$  then
        - ⌄ return( $v$ );
      - ⌃ else if  $\text{Reg}_v[i] = 0$  then  $v := 1-v;$
      - ⌃ if  $v = 1$  then wait( $2i$ )
      - ⌃  $i := i+1;$
  - end