Solutions to Exercise 9

Problem 1.   Note that if we can guarantee that we always have a majority of processes we can use Paxos to solve[1] consensus. Similarly, if we can guarantee that we always have a majority of the memory components we can use Disk Paxos to solve consensus. However, the problems asks us to show that we cannot solve consensus with a majority of the memory or a majority of the processes. In other words, it could be that we have executions where we only have a majority of processes and executions where we only have a majority of memory components. Now, notice that if a process "sees" and uses a majority of processes, it should not wait to use a majority of memory components before deciding, since it could as well be the case that a majority of memory components does not exist. The same argument applies the other way as well, if a process is able to use a majority of memory components, it should not wait for a majority of processes in order to decide. Therefore, the main insight on why we cannot solve consensus in such a setting is the following: Some process might use a majority of processes in order to decide, while some other process might use a majority of memory components, hence we could have processes deciding on different values.

Problem 2.   We can simply use Disk Paxos. We can wait for all the memory components to respond since the correct ones will respond and the failed ones will respond with a NACK. Even if we have $n-1$ memory failures, all the processes would intersect with the remaining single memory component that is correct.

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[1]By solving consensus we mean to satisfy the safety properties of consensus. For the liveness property, some synchrony is also needed.