Problem 1. Given that the splitter will be called concurrently by a number of $N$ threads, we can think about this as selecting 1 thread to return stop. All the threads arriving during this election but not chosen to return stop can return left, and the ones arriving after the election can return right. It is acceptable to not have any threads selected to get stop (e.g., in case more than 1 thread executes splitter), but it must never be possible to have more than 1 thread return stop during a concurrent execution.

We use two registers:

- $P$ (multi-valued), and
- $S$ (binary, initialized to false)

$P$ holds the id of the thread that should get stop. $S$ marks whether a stop thread has been selected. When a thread calls splitter, it needs to check whether $S$ is false, and if so, set it to true and return stop. The difficulty is that we cannot use atomic getAndSet-type primitives, so multiple threads first reading the value of $S$ and then updating it could mistakenly think they each got stop. In order to decide which thread should get stop, each thread volunteers itself by setting the value of $P$ to their own id. The last thread to update $P$ wins.

After volunteering, a thread checks the $S$ flag, and if it is true, then the thread knows it arrived after the election, and so it gets right. If $S$ still false, then the thread (one of potentially many) arrived during the election, so it sets $S$ to true, and checks if it won (i.e., if the value of $P$ is equal to its own id). If the thread won, it simply gets stop. Otherwise, it means some other thread managed to change $P$ after it, hence the current thread lost and gets left.

It is possible that a thread updates $P$ and becomes the winner just as another thread sets $S$ to true, but before checking to see if it won. In this case, 0 threads get stop, as the winner then reads $S$, finds it true, concludes it arrived after the election, and gets right.

However, it is impossible for more than 1 thread to get stop. Assume by way of contradiction that 2 threads with identifiers $i$ and $j$ both return stop. Furthermore, assume without loss of generality that thread $i$ first performed the read of $P$ and then thread $j$ read $P$. Therefore, the order of events will be $\text{read}_i(P = i) \rightarrow \text{read}_j(P = j)$ (i.e., since both threads return stop they read their own identifier when reading from $P$). We furthermore know that both threads write register $P$ at the beginning of their execution and since both threads return stop they read $S$ to be false. So we have the following ordering of events:

- $\text{write}_i(P \leftarrow i) \rightarrow \text{read}_i(S = \text{false}) \rightarrow \text{write}_i(S \leftarrow \text{true}) \rightarrow \text{read}_i(P = i)$.
- $\text{write}_j(P \leftarrow j) \rightarrow \text{read}_j(S = \text{false}) \rightarrow \text{write}_j(S \leftarrow \text{true}) \rightarrow \text{read}_j(P = j)$.

Since thread $i$ read $P = i$ (and thread $j$ read $P = j$) it means that $\text{write}_j(P \leftarrow j)$ takes place after $\text{read}_i(P = i)$. So we have:

- $\text{write}_i(P \leftarrow i) \rightarrow \text{read}_i(S = \text{false}) \rightarrow \text{write}_i(S \leftarrow \text{true}) \rightarrow \text{read}_i(P = i) \rightarrow \text{write}_j(P = j) \rightarrow \text{read}_j(S = \text{false})$.

This is a contradiction, since thread $i$ wrote true to $S$ and then $j$ read false from $S$. 
upon $\text{splitter}_i$

\begin{itemize}
\item $P \leftarrow i$;
\item if $S$ then return "right";
\item $S \leftarrow \text{true}$;
\item if $P = i$ then return "stop";
\item return "left";
\end{itemize}

Algorithm 1: Sample implementation of the $\text{splitter}$ object.
Problem 2.
Algorithm 2 presents the pseudocode of an atomic wait-free snapshot as described in class. For a program running \( N \) threads, in order to run a scan or a collect operation, all the registers of the \( N \) threads need to be read. Writes are done only on a thread’s register \( R[i] \). Since we know beforehand that many of the \( N \) threads will not use the snapshot, a better solution is to assign registers to threads on demand.

We assume that there exists an obtain() operation that each thread can call to get a register that is assigned only to itself. Algorithm 3 presents the implementation of update() and scan() using the aforementioned operation. Importantly, the number of registers that need to be parsed now in scan() is dependent on the number of threads that have written to the object (and thus have been assigned a register).

upon scan

\[
\begin{align*}
  t_1 & \leftarrow \text{collect}() , t_2 \leftarrow t_1; \\
  \textbf{while} & \text{true} \do \\
  t_3 & \leftarrow \text{collect}(); \\
  \text{if} & \; t_3 = t_2 \; \textbf{then} \; \textbf{return} \; \langle t_3[1].val, \ldots, t_3[N].val \rangle; \\
  \textbf{for} & \; k \leftarrow 1 \; \textbf{to} \; N \; \textbf{do} \\
  \text{if} & \; t_3[k].ts \geq t_1[k].ts + 2 \; \textbf{then} \; \textbf{return} \; t_3[k].\text{snapshot}; \\
  t_2 & \leftarrow t_3; \\
\end{align*}
\]

procedure collect()

\[
\begin{align*}
  \textbf{for} & \; k \leftarrow 1 \; \textbf{to} \; N \; \textbf{do} \\
  x[k] & \leftarrow R[k]; \\
  \textbf{return} \; x; \\
\end{align*}
\]

procedure update(v)

\[
\begin{align*}
  ts & \leftarrow ts + 1; \\
  \text{snapshot} & \leftarrow \text{scan}(); \\
  R[i] & \leftarrow \langle ts, v, \text{snapshot} \rangle; \\
\end{align*}
\]

Algorithm 2: Sample implementation of a non-adaptive snapshot. Each thread has its own register.

procedure update(v)

\[
\begin{align*}
  \textbf{if} & \; \text{myreg} = \perp \; \textbf{then} \\
  \text{myreg} & \leftarrow \text{obtain}(); \\
  ts & \leftarrow ts + 1; \\
  \text{snapshot} & \leftarrow \text{scan}(); \\
  R[\text{myreg}] & \leftarrow \langle ts, v, \text{snapshot} \rangle; \\
\end{align*}
\]

upon scan

\[
\begin{align*}
  t_1 & \leftarrow \text{collect}() , t_2 \leftarrow t_1; \\
  \textbf{while} & \text{true} \do \\
  t_3 & \leftarrow \text{collect}(); \\
  \text{if} & \; t_3 = t_2 \; \textbf{then} \; \textbf{return} \; \langle t_3[1].val, \ldots, t_3[t_3.\text{length}].val \rangle; \\
  \textbf{for} & \; k \leftarrow 1 \; \textbf{to} \; t_3.\text{length} \; \textbf{do} \\
  \text{if} & \; t_3[k].ts \geq t_1[k].ts + 2 \; \textbf{then} \; \textbf{return} \; t_3[k].\text{snapshot}; \\
  t_2 & \leftarrow t_3; \\
\end{align*}
\]

Algorithm 3: Sample implementation of update() and scan() in an adaptive snapshot. Each thread that affects the snapshot calls obtain() to get assigned a register.
**Implementing obtain()**

Recall the `splitter` object implemented in the previous exercise: it allows selecting at most 1 thread out of multiple accessing the object concurrently, while partitioning the remaining threads into 2 separate pools (`left`, `right`). Keeping this in mind, we create a matrix of registers and `splitter`, as presented in figure 1. A thread calling `obtain()` starts from the top-left corner and calls the `splitter` in that cell. If it gets `stop`, then it obtains that register. Otherwise, it moves 1 column to the `right`, or 1 row downwards for `left`, and repeats the process.

```plaintext
procedure obtain()
    x ← 1, y ← 1;
    while true do
        s ← S[x, y].splitter();
        if s = "stop" then return R[x, y];
        else if s = "left" then y ← y + 1;
        else x ← x + 1;
    end while
end procedure
```

Algorithm 4: Implementation of `obtain()` using a matrix of registers and `splitter` objects.

**Implementing collect()**

Finally, we need to adapt the `collect()` call to the matrix of registers now being used. The insight here is that all the registers that have been assigned from each matrix diagonal that has had at least 1 splitter used need to be taken into account.

```plaintext
procedure collect()
    C ← ⟨⟩;
    d ← 1;
    while diagonal d has a splitter that has been traversed do
        C ← C · ⟨values of all non-⊥ registers on diagonal d⟩;
        d ← d + 1;
    end while
    return C;
end procedure
```

Algorithm 5: Implementation of `collect()` using a matrix of registers and `splitter` objects.