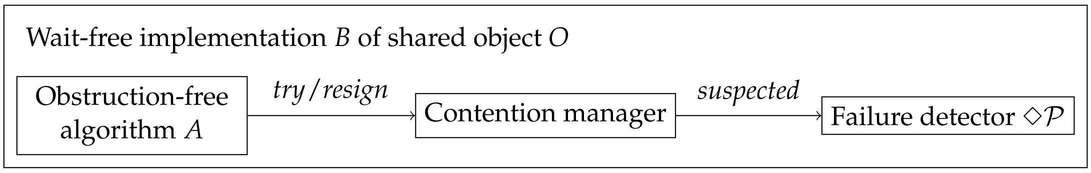
Concurrent Algorithms

Solutions to Exercise 6

Problem 1.

Let A be an *obstruction-free* algorithm implementing some shared object O with operations $op_1, \ldots, op_k$. The goal of the exercise is to transform algorithm A into a *wait-free* algorithm B that also implements shared object O (i.e., the operations $op_1, \ldots, op_k$). We will do it by implementing an abstraction called a *contention manager*, using an *eventually perfect* failure detector $\diamond\mathcal{P}$ and atomic registers.



A contention manager implements two operations: try_i and $resign_i$ (invoked by process p_i). These operations do not take any arguments and always return *ok*. A contention manager resolves contention, and thus guarantees wait-freedom, by delaying some processes that have invoked try_i. In other words, when a process p_i invokes try_i, a contention manager can decide when to return from the operation—it can delay the response of try_i for an arbitrarily long time.

We assume that algorithm A uses the interface of the contention manager, i.e., that it invokes try_i and $resign_i$. More precisely, every time an operation op_m, implemented by A, is executed by a process p_i, the following conditions are satisfied:

1. try_i is called always before the first step of the implementation of op_m is executed (i.e., just after op_m is invoked), and possibly many times while op_m is being executed, (You may stop the implementation of op_m at some point, call try_i, and later resume op_m at the same point.)
2. $resign_i$ is called *only* immediately after the last step of the implementation of op_m is executed (i.e., just before the result of op_m is returned),
3. If process p_i is correct but does not return from operation op_m (i.e., the implementation of the operation keeps executing), then p_i keeps calling try_i many times. (The number of times should be finite as the problem asks you for a wait-free algorithm. However, the number is unbounded as the failure detector introduced below only guarantees some property after some unknown time.)

Moreover, every time process p_i invokes try_i or $resign_i$, p_i waits until $try_i/resign_i$ returns before executing any further steps of algorithm A.

An eventually perfect failure detector $\diamond\mathcal{P}$ maintains, at every process p_i, a set $suspected_i$ of suspected processes. $\diamond\mathcal{P}$ guarantees that eventually, after some unknown time, the following conditions are satisfied:

1. Every correct process permanently suspects every crashed process,
2. No correct process is ever suspected by any correct process.

This means that $suspected_i$ can be arbitrary and different at every process for any *finite* period of time. However, eventually, at every correct process p_i, set $suspected_i$ will be permanently equal to the set of processes that have crashed.

Your task is to implement a contention manager C (i.e., the operations try_i and $resign_i$, for every process p_i) that converts obstruction-free algorithm A into wait-free algorithm B, and that uses only atomic registers and failure detector $\diamond\mathcal{P}$.

Solution

The following algorithm implements a contention manager that transforms any obstruction-free algorithm into a wait-free one:

uses: $T[1, \ldots, N]$—array of registers
initially: $T[1, \ldots, N] \leftarrow \bot$

upon try_i **do**
  if $T[i] = \bot$ **then** $T[i] \leftarrow$ `GetTimestamp()`

  repeat
    $sact_i \leftarrow \{\, p_j \mid T[j] \neq \bot \wedge p_j \notin \Diamond\mathcal{P}.suspected_i \,\}$
    $leader_i \leftarrow$ the process in $sact_i$ with the lowest timestamp $T[leader_i]$
  until $leader_i = p_i$
  return *ok*

upon $resign_i$ **do**
  $T[i] \leftarrow \bot$
  return *ok*

The algorithm uses a procedure `GetTimestamp()` that generates *unique* timestamps. We assume that if a process gets a timestamp t from `GetTimestamp()`, then no process can get a timestamp lower than t infinitely many times. Such a procedure can be implemented as follows, using only registers.

uses: $R[1, \ldots, N]$—array of registers
initially: $R[1, \ldots, N] \leftarrow 0$

upon $GetTimeStamp_i$ **do**
  $temp_i \leftarrow R[i] + 1$
  $R[i] \leftarrow temp_i$
  $sum_i \leftarrow 0$
  for $j = 1$ *to* N **do**
    $sum_i \leftarrow sum_i + R[j]$
  return (i, sum_i)

Then to find a lowest timestamp, we define an order between two pairs (i, t_1) and (j, t_2) as follows: $(i, t_1) < (j, t_2)$ if $t_1 < t_2$, or $t_1 = t_2$ and $i < j$.

We note that a process may invoke try_i many times until it finds the implementation of algorithm A for operation op_m terminates. This is due to the fact that the failure detector is only *eventually* perfect, which can make mistakes in suspecting processes in some finite period of time. Thus after the first try_i returns ok, p_i may be in fact running op_m concurrently with another process and takes an infinite number of steps (since A is only obstruction-free). However, we can avoid this by looking into the implementation of A for op_m, stop op_m from time to time, invoke try_i again and resume op_m; we may repeat this until op_m finishes.