Concurrent Algorithms November 1, 2021

Solutions to Exercise 4

Problem 1. Given that the *splitter* will be called concurrently by a number of N threads, we can think about this as selecting 1 thread to return *stop*. All the threads arriving during this election but not chosen to return *stop* can return *left*, and the ones arriving after the election can return *right*. It is acceptable to not have any threads selected to get *stop* (e.g., in case more than 1 thread executes *splitter*), but it must never be possible to have more than 1 thread return *stop* during a concurrent execution.

We use two registers:

- P (multi-valued), and
- S (binary, initialized to *false*)

P holds the *id* of the thread that should get *stop*. S marks whether a *stop* thread has been selected. When a thread calls *splitter*, it needs to check whether S is *false*, and if so, set it to *true* and return *stop*. The difficulty is that we cannot use atomic *getAndSet*-type primitives, so multiple threads first reading the value of S and then updating it could mistakenly think they each got *stop*. In order to decide which thread should get *stop*, each thread volunteers itself by setting the value of P to their own id. The last thread to update P wins.

After volunteering, a thread checks the S flag, and if it is *true*, then the thread knows it arrived after the election, and so it gets *right*. If S still *false*, then the thread (one of potentially many) arrived during the election, so it sets S to *true*, and checks if it won (i.e., if the value of P is equal to its own id). If the thread won, it simply gets *stop*. Otherwise, it means some other thread managed to change P after it, hence the current thread lost and gets *left*.

It is possible that a thread updates P and becomes the winner just as another thread sets S to *true*, but before checking to see if it won. In this case, 0 threads get *stop*, as the winner then reads S, finds it *true*, concludes it arrived after the election, and gets *right*.

However, it is impossible for more than 1 thread to get *stop*. Assume by way of contradiction that 2 threads with identifiers i and j both return *stop*. Furthermore, assume without loss of generality that thread i first performed the read of P and then thread j read P. Therefore, the order of events will be $read_i(P = i) \rightarrow read_j(P = j)$ (i.e., since both threads return *stop* they read their own identifier when reading from P). We furthermore know that both threads write register P at the beginning of their execution and since both threads return *stop* they read S to be *false*. So we have the following ordering of events:

- $write_i(P \leftarrow i) \rightarrow read_i(S = false) \rightarrow write_i(S \leftarrow true) \rightarrow read_i(P = i)$.
- $write_j(P \leftarrow j) \rightarrow read_j(S = false) \rightarrow write_j(S \leftarrow true) \rightarrow read_j(P = j)$.

Since thread i read $P = i$ (and thread j read $P = j$) it means that $write_j(P \leftarrow j)$ takes place after $read_i(P = i)$. So we have:

- $write_i(P \leftarrow i) \rightarrow read_i(S = false) \rightarrow write_i(S \leftarrow true) \rightarrow read_i(P = i) \rightarrow write_j(P = j) \rightarrow read_j(S = false)$.

This is a contradiction, since thread i wrote *true* to S and then j read *false* from S.

upon *splitter*$_i$

$P \leftarrow i$;

if S **then return** *"right"*;

$S \leftarrow$ *true*;

if $P = i$ **then return** *"stop"*;

return *"left"*;

Algorithm 1: Sample implementation of the *splitter* object.

Problem 2.

Algorithm 2 presents the pseudocode of an atomic wait-free snapshot as described in class. For a program running N threads, in order to run a *scan* or a *collect* operation, all the registers of the N threads need to be read. Writes are done only on a thread's register $R[i]$. Since we know beforehand that many of the N threads will not use the snapshot, a better solution is to assign registers to threads on demand.

We assume that there exists an *obtain*() operation that each thread can call to get a register that is assigned only to itself. Algorithm 3 presents the implementation of *update*() and *scan*() using the aforementioned operation. Importantly, the number of registers that need to be parsed now in *scan*() is dependent on the number of threads that have written to the object (and thus have been assigned a register).

upon $scan_i$
  $t_1 \leftarrow collect(), t_2 \leftarrow t_1$;
  while *true* **do**
    $t_3 \leftarrow collect()$;
    if $t_3 = t_2$ **then return** $\langle t_3[1].val, \ldots, t_3[\mathbf{N}].val \rangle$;
    for $k \leftarrow 1$ **to** N **do**
      if $t_3[k].ts \geq t_1[k].ts + 2$ **then return** $t_3[k].snapshot$;
    $t_2 \leftarrow t_3$;

procedure $collect()$
  for $k \leftarrow 1$ **to** N **do**
    $x[k] \leftarrow R[k]$;
  return x;

procedure $update_i(v)$
  $ts \leftarrow ts + 1$;
  $snapshot \leftarrow scan()$;
  $R[\mathbf{i}] \leftarrow \langle ts, v, snapshot \rangle$;

Algorithm 2: Sample implementation of a non-adaptive snapshot. Each thread has its own register.

procedure $update(v)$
  if $myreg = \perp$ **then**
    $\boldsymbol{myreg \leftarrow obtain();}$
  $ts \leftarrow ts + 1$;
  $snapshot \leftarrow scan()$;
  $R[\boldsymbol{myreg}] \leftarrow \langle ts, v, snapshot \rangle$;

upon $scan_i$
  $t_1 \leftarrow collect(), t_2 \leftarrow t_1$;
  while *true* **do**
    $t_3 \leftarrow collect()$;
    if $t_3 = t_2$ **then return** $\langle t_3[1].val, \ldots, t_3[t_3.\boldsymbol{length}].val \rangle$;
    for $k \leftarrow 1$ **to** $t_3.\boldsymbol{length}$ **do**
      if $t_3[k].ts \geq t_1[k].ts + 2$ **then return** $t_3[k].snapshot$;
    $t_2 \leftarrow t_3$;

Algorithm 3: Sample implementation of *update*() and *scan*() in an adaptive snapshot. Each thread that affects the snapshot calls *obtain*() to get assigned a register.

Implementing *obtain*()

Recall the *splitter* object implemented in the previous exercise: it allows selecting at most 1 thread out of multiple accessing the object concurrently, while partitioning the remaining threads into 2 separate pools (*left*, *right*). Keeping this in mind, we create a matrix of *registers* and *splitters*, as presented in figure 1. A thread calling *obtain*() starts from the top-left corner and calls the *splitter* in that cell. If it gets *stop*, then it obtains that register. Otherwise, it moves 1 column to the *right*, or 1 row downwards for *left*, and repeats the process.

procedure *obtain*()
  $x \leftarrow 1, y \leftarrow 1$;
  while *true* **do**
    $s \leftarrow S[x, y].splitter()$;
    if $s =$ "*stop*" **then return** $R[x, y]$;
    else if $s =$ "*left*" **then** $y \leftarrow y + 1$;
    else $x \leftarrow x + 1$;

Algorithm 4: Implementation of *obtain*() using a matrix of registers and *splitter* objects.

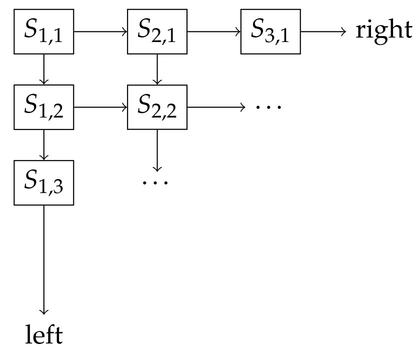

Figure 1: Matrix of registers and *splitter* objects.

Implementing *collect*()

Finally, we need to adapt the *collect*() call to the matrix of registers now being used. The insight here is that all the registers that have been assigned from each matrix diagonal that has had <u>at least</u> 1 splitter used need to be taken into account.

procedure *collect*()
  $C \leftarrow \langle\,\rangle$;
  $d \leftarrow 1$;
  while *diagonal d has a splitter that has been traversed* **do**
    $C \leftarrow C \cdot \langle$ values of all non-$\perp$ registers on diagonal $d\,\rangle$;
    $d \leftarrow d + 1$;
  return C;

Algorithm 5: Implementation of *collect*() using a matrix of registers and *splitter* objects.

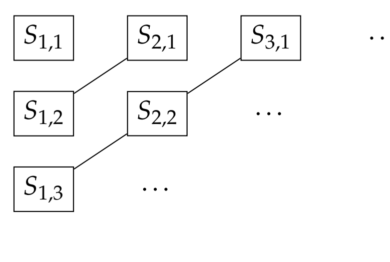

Figure 2: Matrix of registers and *splitter* objects.