Distributed systems

Consensus

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Consensus

• In the consensus problem, the processes propose values and have to agree on one among these values.

• Solving consensus is key to solving many problems in distributed computing (e.g., total order broadcast, atomic commit, terminating reliable broadcast).
Consensus

C1. Validity: Any value decided is a value proposed

C2. Agreement: No two correct processes decide differently

C3. Termination: Every correct process eventually decides

C4. Integrity: No process decides twice
Consensus
Uniform consensus

**C1. Validity:** Any value decided is a value proposed

**C2’. Uniform Agreement:** No two processes decide differently

**C3. Termination:** Every correct process eventually decides

**C4. Integrity:** No process decides twice
Uniform consensus

- Propose(0)
- Decide(0)
- Propose(1)
- Decide(0)
- Crash
- Propose(0)
- Decide(0)
- Propose(0)
Consensus

Events

• Request: \(<\text{Propose, v}>\)
• Indication: \(<\text{Decide, v’}>\)

Properties:
• \(C_1, C_2, C_3, C_4\)
Modules of a process

Applications

Consensus

(R-U) Reliable broadcast

Failure detector

Channels

decide

propose

broadcast

deliver

suspect
Consensus algorithm I

- A P-based (fail-stop) consensus algorithm

- The processes exchange and update proposals in rounds and decide on the value of the non-suspected process with the smallest id
Consensus algorithm II

• A P-based (i.e., fail-stop) uniform consensus algorithm

• The processes exchange and update proposal in rounds, and after n rounds decide on the current proposal value [Lyn96]
Consensus algorithm III

• A <>P-based uniform consensus algorithm assuming a correct majority

• The processes alternate in the role of a coordinator until one of them succeeds in imposing a decision [DLS88,CT91]
Consensus algorithm I

- The processes go through rounds incrementally (1 to n): in each round, the process with the id corresponding to that round is the leader of the round.
- The leader of a round decides its current proposal and broadcasts it to all.
- A process that is not leader in a round waits (a) to deliver the proposal of the leader in that round to adopt it, or (b) to suspect the leader.
Consensus algorithm I

**Implements:** Consensus (cons).

**Uses:**
- BestEffortBroadcast (beb).
- PerfectFailureDetector (P).

**upon event** < Init > **do**
- suspected := ∅;
- round := 1; currentProposal := nil;
- broadcast := delivered[] := false;
Consensus algorithm I

\[ \text{upon event} < \text{crash, pi} > \text { do} \]
\[ \text{suspected} := \text{suspected} \cup \{\text{pi}\}; \]

\[ \text{upon event} < \text{Propose, v} > \text { do} \]
\[ \text{if currentProposal} = \text{nil} \text { then} \]
\[ \text{currentProposal} := \text{v}; \]
Consensus algorithm I

\begin{itemize}
\item upon event \(<\text{bebDeliver}, p_{\text{round}}, \text{value}>\) do
  \begin{itemize}
  \item currentProposal := value;
  \item delivered[\text{round}] := true;
  \end{itemize}
\end{itemize}

\begin{itemize}
\item upon event \(\text{delivered}[\text{round}] = \text{true or } p_{\text{round}} \in \text{suspected}\) do
  \begin{itemize}
  \item round := round + 1;
  \end{itemize}
\end{itemize}
Consensus algorithm I

\[\text{upon event } p_{\text{round}} = \text{self and broadcast} = \text{false and currentProposal} \neq \text{nil do}
\]

\[\text{trigger} < \text{Decide, currentProposal}>;\]

\[\text{trigger} < \text{bebBroadcast, currentProposal}>;\]

\[\text{broadcast} := \text{true};\]
Consensus algorithm I

propose(0)  decide(0)

p1

propose(1)

p2

propose(0)

p3

decide(0)
Consensus algorithm I

- propose(0) at p1
- propose(1) at p2
- propose(0) at p3

- decide(0)
- decide(1)
- crash

- decide(1)
Correctness argument

• Let $p_i$ be the correct process with the smallest id in a run $R$.

• Assume $p_i$ decides $v$.
  • If $i = n$, then $p_n$ is the only correct process.
  • Otherwise, in round $i$, all correct processes receive $v$ and will not decide anything different from $v$. 
Consensus algorithm II

• Algorithm II implements uniform consensus
• The processes go through rounds incrementally (1 to n): in each round I, process $p_1$ sends its currentProposal to all.
• A process adopts any currentProposal it receives.
• Processes decide on their currentProposal values at the end of round n.
Consensus algorithm II

**Implements:** Uniform Consensus (ucons).

**Uses:**
- BestEffortBroadcast (beb).
- PerfectFailureDetector (P).

- **upon event** `< Init >` **do**
  - suspected := ∅;
  - round := 1; currentProposal := nil;
  - broadcast := delivered[] := false;
  - decided := false
Consensus algorithm II

\textbf{upon event \langle crash, pi \rangle do}
\begin{align*}
\text{suspected} & := \text{suspected} \cup \{pi\}; \\
\end{align*}

\textbf{upon event \langle Propose, v \rangle do}
\begin{align*}
\text{if currentProposal} & = \text{nil then} \\
\text{currentProposal} & := v;
\end{align*}
Consensus algorithm II

Upon event < bebDeliver, $p_{\text{round}}$, value > do

- currentProposal := value;
- delivered[round] := true;
Consensus algorithm II

\begin{verbatim}
\textbf{upon event} \ \text{delivered[round]} = \text{true} \ \textbf{or} \ \ \text{p}_{\text{round}} \in \text{suspected} \ \textbf{do}

\textbf{if} \ \text{round=}\text{n} \ \textbf{and} \ \text{decided=false} \ \textbf{then}

\textbf{trigger} \ <\text{Decide, currentProposal}>

\text{decided=}\text{true}

\textbf{else}

\text{round} := \text{round} + 1
\end{verbatim}
Consensus algorithm II

upon event \( p_{\text{round}} = \text{self} \) and broadcast = false and currentProposal \( \neq \) nil do

- trigger <bebBroadcast, currentProposal>;
- broadcast := true;
Consensus algorithm II

propose(0)

p1

propose(1)

p2

propose(0)

p3

decide(0)

decide(0)

decide(0)
Correctness argument (A)

• **Lemma:** If a process $p_J$ completes round $I$ without receiving any message from $p_I$ and $J > I$, then $p_I$ crashes by the end of round $J$.

• Proof: Suppose $p_J$ completes round $I$ without receiving a message from $p_I$, $J > I$, and $p_I$ completes round $J$. Since $p_J$ suspects $p_I$ in round $I$, $p_I$ has crashed before $p_J$ completes round $I$. In round $J$ either $p_I$ suspects $p_J$ (not possible because $p_I$ crashes before $p_J$) or $p_I$ receives round $J$ message from $p_J$ (also not possible because $p_I$ crashes before $p_J$ completes round $I < J$).
Correctness argument (B)

• **Uniform agreement:** Consider the process with the lowest id which decides, say $p_I$. Thus, $p_I$ completes round $n$. By our previous lemma, in round $I$, every $p_J$ with $J > I$ receives the currentProposal of $p_I$ and adopts it. Thus, every process which sends a message after round $I$ or decides, has the same currentProposal at the end of round $I$. 
Consensus algorithm III

• A uniform consensus algorithm assuming:
  • a correct majority
  • a $<>P$ failure detector

• Basic idea: the processes alternate in the role of a phase coordinator until one of them succeeds in imposing a decision
Consensus algorithm III

• $<>P$ ensures:

  • *Strong completeness*: eventually every process that crashes is permanently suspected by all correct processes

  • *Eventual strong accuracy*: eventually no correct process is suspected by any process
"<>" makes a difference

• *Eventual strong accuracy:* strong accuracy holds only *after finite time.*

• Correct processes may be *falsely suspected* a finite number of times.

• This breaks consensus algorithms I and II
  • Counter examples for algorithm I and II (see next slide)
Agreement violated with $<>P$ in algorithm I

- $p_1$: propose(0) → decide(0) → suspect $p_1$
- $p_2$: propose(1) → decide(1)
- $p_3$: propose(1)
Agreement violated with $<>P$ in algorithm II

- $p_1$ proposes 0
- $p_2$ proposes 1
- $p_3$ proposes 1
- $p_1$ suspects $p_2$
- $p_2$ suspects $p_1$
- $p_3$ suspects $p_2$
- $p_3$ suspects $p_3$
- Decide 0
- Decide 1
- Decide 1
- Decide 1
Consensus algorithm III

• The algorithm is also round-based: processes move incrementally from one round to the other

• Process $\pi_i$ is **leader** in every round $k$ such that $k \mod N = i$

• In such a round, $\pi_i$ **tries to decide** (next 2 slides)
Consensus algorithm III

• pi succeeds if it is not suspected (processes that suspect pi inform pi and move to the next round; pi does so as well)

• If pi succeeds, pi uses a reliable broadcast to send the decision to all (the reliability of the broadcast is important here to preclude the case where pi crashes, some other processes delivers the message and stop while the rest keeps going without the majority)
Consensus algorithm III

• To decide, pi executes steps 1-2-3
  
  • 1. pi selects among a majority the latest adopted value (latest with respect to the round in which the value is adopted – see step 2)
  
  • 2. pi imposes that value at a majority: any process in that majority adopts that value – pi fails if it is suspected
  
  • 3. pi decides and broadcasts the decision to all
Consensus algorithm III

P1
propose(0) p1’s round p2’s round p3’s round p1’s round etc
propose(1)

P2
propose(0)

P3
Consensus algorithm III

P1
propose(0)
propose(1)
propose(0)

P2
[1] [0]
[0] [0]
[0] [0]

P3
[0] [0]
[0] [0]
[0] [0]

step 1
step 2
step 3

round 1

decide(0)
decide(0)
decide(0)
Consensus algorithm III

- P1: propose(0) → crash → propose(1)
- P2: propose(0) → nack
- P3: propose(0) → nack
Consensus algorithm III

Propose(0) propose(1) propose(1)

P1 P2 P3

p1's round p2's round p3's round p1's round etc

0 1 0

nack
Correctness argument A

• **Validity** and **integrity** are trivial

• Consider **termination**: if a correct process decides, it uses reliable broadcast to send the decision to all: every correct process decides

• Assume by contradiction that some process is correct and no correct process decides. We argue that this is impossible.
Correctness argument A’

• By the correct *majority* assumption and the *completeness* property of the failure detector, no correct process remains blocked forever in some phase.

• By the *accuracy* property of the failure detector, some correct process reaches a round where it is leader and it is not suspected and reaches a decision in that round: a contradiction
Correctness argument B

• Consider now agreement

• Let k be the first round in which some process pi decides some value v, i.e., pi is the leader of round k and pi decides v in k

• This means that, in round k, a majority of processes have adopted v

• By the algorithm, no value else than v will be proposed (and hence decided) by any process in a round higher than k
Correctness argument B

new

\[ p_1 \]
\[ n/2 \]
\[ p_n \]

round \( k \)

\[ \nu \]

\[ \text{acks}, \text{decide}(\nu) \]

round \( k+1 \)

\[ \nu \]

\[ \text{impose } \nu \]

\[ \text{gather} \]

\[ = \text{leader of that round} \]
Agreement is never violated

- Look at a "totally unreliable" failure detector (provides no guarantees)
  - may always suspect everybody
  - may never suspect anybody
- Agreement is not violated
  - Can use the same correctness argument as before
  - Termination not ensured (everybody may be suspected infinitely often)
Summary

• (Uniform) Consensus problem is an important problem to maintain consistency
• Three algorithms:
  • I: consensus using P
  • II: uniform consensus using P
  • III: uniform consensus using <>P and a correct majority