

## Exercise Session 7

### Terminating Reliable Broadcast

#### Problem 1

Can we implement TRB with the eventually perfect failure detector  $\diamond P$ , if we assume that at least one process can crash?

The answer is no. Consider an instance  $trb$  of TRB with sender process  $s$ . We show that it is impossible to implement TRB from an eventually perfect failure-detector primitive  $\diamond P$ , if even one process can crash.

Consider an execution  $E_1$ , in which process  $s$  crashes initially and observe the possible actions for some correct process  $p$ : due to the termination property of TRB, there must be a time  $T$  at which  $p$   $trb$ -delivers  $\Delta$ .

Consider a second execution  $E_2$  that is similar to  $E_1$  up to time  $T$ , except that the sender  $s$  is correct and  $trb$ -broadcasts some message  $m$ , but all communication messages to and from  $s$  are delayed until after time  $T$ . The failure detector behaves in  $E_2$  as in  $E_1$  until after time  $T$ . This is possible because the failure detector is only eventually perfect. Up to time  $T$ , process  $p$  cannot distinguish  $E_1$  from  $E_2$  and  $trb$ -delivers. According to the *agreement* property of TRB, process  $s$  must  $trb$ -deliver as well, and  $s$  delivers exactly one message due to the *termination* property. But this contradicts the *validity* property of TRB, since  $s$  is correct, has  $trb$ -broadcast some message  $m \neq \Delta$ , and must  $trb$ -deliver  $m$ .

#### Problem 2

Show that  $P$  is the weakest failure detector for Group Membership.

In order to show that  $P$  is the weakest failure detector for Group Membership, we need to show that:

- $P$  can be used to implement Group Membership.
- Group Membership can be used to implement  $P$ .

The first direction stems directly from the Group Membership implementation in the class.

For the second direction, we assume that all processes run Group Membership algorithm. Whenever a new view is installed, all processes that are freshly removed from the view are added to the suspected set. This approach satisfies both *Strong Completeness* and *Strong Accuracy* of  $P$ , directly from the corresponding properties of Group Membership.