Distributed Algorithms

Fall 2020

Reliable & Causal Broadcast - solutions
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Reliable broadcast

Specification:

- **Validity**: If a *correct* process broadcasts *m*, then it eventually delivers *m*.
- **Integrity**: m is delivered by a process at most once, and only if it was previously broadcast.
- **Agreement**: If a correct process delivers *m*, then all correct processes eventually deliver *m*.

Algorithm: Lazy Reliable Broadcast

Implements:
ReliableBroadcast, **instance** *rb*.

Uses:
BestEffortBroadcast, **instance** *beb*;
PerfectFailureDetector, **instance** $\mathcal{P}$.

upon event ⟨ *rb*, *Init* ⟩ **do**
correct := Π;
from[p] := $[\emptyset]^N$;

upon event ⟨ *rb*, *Broadcast* | m ⟩ **do**
trigger ⟨ *beb*, *Broadcast* | [DATA, *self*, m] ⟩;

upon event ⟨ *beb*, *Deliver* | p, [DATA, s, m] ⟩ **do**
if $m \notin$ *from*[s] **then**
trigger ⟨ *rb*, *Deliver* | s, m ⟩;
from[s] := *from*[s] $\cup \{m\}$;
if $s \notin$ *correct* **then**
trigger ⟨ *beb*, *Broadcast* | [DATA, s, m] ⟩;

upon event ⟨ $\mathcal{P}$, *Crash* | p ⟩ **do**
correct := *correct* $\setminus \{p\}$;
forall $m \in$ *from*[p] **do**
trigger ⟨ *beb*, *Broadcast* | [DATA, p, m] ⟩;

Strong accuracy:
No correct process is ever suspected:

$$\forall F, \forall H, \forall t \in \mathcal{T}, \forall p \in correct(F), \forall q : p \notin H(q,t)$$

Strong completeness:
Eventually, every faulty process is permanently suspected by every correct process:

$$\forall F, \forall H, \exists t \in \mathcal{T}, \forall p \in crashed(F), \forall q \in correct(F), \forall t' \geq t : p \in H(q,t')$$

Where:

- crashed(F) is the set of crashed processes.
- correct(F) is the set of correct processes.
- H(p, t) is the output of the failure detector of process p at time t.

Exercise 1

Implement a reliable broadcast algorithm without using any failure detector, i.e., using only *BestEffort-Broadcast(BEB)*.

Exercise 1 (Solution)

Use a step of all-to-all communication.

In particular, very process that gets a message relays it immediately.

Recall that in the original algorithm, processes were relaying messages from a process p only if p crashes.

```
upon initialization do
        delivered := {}

upon RB-broadcast(m) do
        send(m) to Π \ {p}
        RB-deliver(m)

upon BEB-receive(m) from q do
        if not m ∈ delivered
                send (m) to Π \ {p, q}
                RB-deliver(m)
                delivered := delivered ∪ m
```

Agreement: Before RB-delivering m, a correct process p forwards m to all processes. By the properties of perfect channels and the fact that p is correct, all correct processes will eventually receive m and RB-deliver it.

Exercise 2

The reliable broadcast algorithm presented in class has the processes continuously fill their different buffers without emptying them.

```
Implements: ReliableBroadcast (rb).
Uses:
    BestEffortBroadcast (beb).
    PerfectFailureDetector (P).
upon event < Init > do
    delivered := ∅;
    correct := S;
    forall pi ∈ S do from[pi] := ∅;
```

```
upon event < rbBroadcast, m> do
    delivered := delivered U {m};
    trigger < rbDeliver, self, m>;
    trigger < bebBroadcast, [Data,self,m]>;

upon event < crash, pi > do
    correct := correct \ {pi};
    forall [pj,m] ∈ from[pi] do
        trigger < bebBroadcast,[Data,pj,m]>;
```

```
upon event < bebDeliver, pi, [Data,pj,m]> do
    if m ∉ delivered then
        delivered := delivered U {m};
        trigger < rbDeliver, pj, m>;
        if pi ∉ correct then
            trigger < bebBroadcast,[Data,pj,m]>;
        else
            from[pi] := from[pi] U {[pj,m]};
```

Modify it to remove (i.e. garbage collect) unnecessary messages from the buffers:

A. *from*, and
B. *delivered*

Exercise 2 (Solution)

A. The *from* buffer is used only to store messages that are relayed in the case of a failure. Therefore, messages from the *from* buffer can be removed as soon as they are relayed.

B. Messages from the *delivered* array cannot be removed. Consider this scenario: If a process crashes and its messages are retransmitted by two different processes, then a process might RB-deliver the same message twice if it empties the *delivered* buffer in the meantime. This is a violation of the “no duplication” property.

Uniform reliable broadcast

Specification:

- **Validity**: If a *correct* process broadcasts *m*, then it eventually delivers *m*.
- **Integrity**: m is delivered by a process at most once, and only if it was previously broadcast.
- **Uniform Agreement**: If a ~~correct~~ process delivers *m*, then all correct processes eventually deliver *m*.

Algorithm: All-Ack Uniform Reliable Broadcast

```
Implements:
    UniformReliableBroadcast, instance urb.

Uses:
    BestEffortBroadcast, instance beb.
    PerfectFailureDetector, instance P.

upon event ⟨ urb, Init ⟩ do
    delivered := ∅;
    pending := ∅;
    correct := Π;
    forall m do ack[m] := ∅;

upon event ⟨ urb, Broadcast | m ⟩ do
    pending := pending ∪ {(self, m)};
    trigger ⟨ beb, Broadcast | [DATA, self, m] ⟩;

upon event ⟨ beb, Deliver | p, [DATA, s, m] ⟩ do
    ack[m] := ack[m] ∪ {p};
    if (s, m) ∉ pending then
        pending := pending ∪ {(s, m)};
        trigger ⟨ beb, Broadcast | [DATA, s, m] ⟩;

upon event ⟨ P, Crash | p ⟩ do
    correct := correct \ {p};

function candeliver(m) returns Boolean is
    return (correct ⊆ ack[m]);

upon exists (s, m) ∈ pending such that candeliver(m) ∧ m ∉ delivered do
    delivered := delivered ∪ {m};
    trigger ⟨ urb, Deliver | s, m ⟩;
```

Exercise 3

What happens in the reliable broadcast and uniform reliable broadcast algorithms if the:

A. accuracy, or
B. completeness

property of the failure detector is violated?

Exercise 3 (Solution 1/2)

Reliable broadcast:

1. Suppose that accuracy is violated. Then, the processes might be relaying messages when this is not really necessary. This wastes resource, but does not impact correctness.
2. Suppose that completeness is violated. Then, the processes might not be relaying messages they should be relaying. This may violate agreement. For instance, assume that only a single process p_1 BEB-delivers (hence RB-delivers) a message m from a crashed process p_2. If a failure detector (at p_1) does not ever suspect p_2, no other correct process will deliver m (agreement is violated).

Exercise 3 (Solution 2/2)

Uniform Reliable broadcast:

Consider a system of three processes p_1, p_2 and p_3. Assume that p_1 URB-broadcasts the message m.

1. Suppose that accuracy is violated. Assume that p_1 falsely suspects p_2 and p_3 to have crashed. p_1 eventually URB-delivers m. Assume that p_1 crashes afterwards. It may happen that p_2 and p_3 never BEB-deliver m and have no knowledge about m (uniform agreement is violated).
2. Suppose that completeness is violated. p_1 might never URB-deliver m if either p_2 or p_3 crashes and p_1 never detects their crash. Hence, p_1 would wait indefinitely for p_2 and p_3 to relay m (validity property violation)

Exercise 4

Implement a **uniform** reliable broadcast algorithm without using any failure detector, i.e., using only *BestEffort-Broadcast(BEB)*.

Exercise 4 (Solution)

Just modify the “candeliver” function.

```
Function candeliver(m) returns Boolean is
        return #(ack[m]) > N / 2
```

Uniform agreement:

Suppose that a correct process delivers m. That means that at least one correct process p “acknowledged” m (rebroadcast m using BestEffortBroadcast). Consequently, all correct processes eventually deliver m from BestEffortBroadcast broadcast by p and rebroadcast m themselves (if they have not done that yet). Hence, every correct process eventually collects at least N/2 acknowledgements and delivers m.

Causal Broadcast

Definition (Happens-before):

We say that an event *e* happens-before an event *e'*, and we write *e* → *e'*, if one of the following three cases holds (is true):

$$\exists p_i \in \Pi \; s.t. \; e = e_i^r, \; e' = e_i^s, \; r < s$$

(*e* and *e'* are executed by the same process)

$$e = send(m, *) \land e' = receive(m)$$

(*e* and *e'* are send/receive events of a message respectively)

$$\exists e'' \; s.t. \; e \rightarrow e'' \rightarrow e'$$

(i.e. → is transitive)

Causal Broadcast

Specification:

It has the same specification of reliable broadcast, with the additional ordering constraint of causal order.

More precisely (causal order):

$$broadcast_p(m) \rightarrow broadcast_q(m') \Rightarrow deliver_r(m) \rightarrow deliver_r(m')$$

Which means that:
If the broadcast of a message *m happens-before* the broadcast of a message *m'*, then no process delivers *m'* unless it has previously delivered *m*.

Exercise 5

Can we devise a broadcast algorithm that does **not** ensure the causal delivery property **but only** (in) its non-uniform variant:

No correct process p_i delivers a message m_2 unless p_i has already delivered every message m_1 such that $m_1 \rightarrow m_2$?

Exercise 5 (Solution)

No! Assume that some algorithm does not ensure the causal delivery property but ensures its non-uniform variant. Assume also that $m_1 \rightarrow m_2$.

This means that a correct process has to deliver m_1 before delivering m_2, but a faulty process is allowed to deliver m_2 and not deliver m_1.

However, a process doesn't know that is faulty in advance (i.e., before it crashes). So, no algorithm can "treat faulty processes in a special way", i.e., a process has to behave correctly until it crashes.

Reminder (Causal delivery property): For any message m_1 that potentially caused a message m_2 , i.e., m1 → m2, no process delivers m_2 unless it has already delivered m_1.

Exercise 6

Suggest a memory optimization of the garbage collection scheme of the following algorithm:

No-Waiting Causal Broadcast

Implements:
CausalOrderReliableBroadcast, **instance** *crb*.

Uses:
ReliableBroadcast, **instance** *rb*.

```
upon event ⟨ crb, Init ⟩ do
    delivered := ∅;
    past := [];

upon event ⟨ crb, Broadcast | m ⟩ do
    trigger ⟨ rb, Broadcast | [DATA, past, m] ⟩;
    append(past, (self, m));

upon event ⟨ rb, Deliver | p, [DATA, mpast, m] ⟩ do
    if m ∉ delivered then
        forall (s, n) ∈ mpast do          // by the order in the list
            if n ∉ delivered then
                trigger ⟨ crb, Deliver | s, n ⟩;
                delivered := delivered ∪ {n};
                if (s, n) ∉ past then
                    append(past, (s, n));
        trigger ⟨ crb, Deliver | p, m ⟩;
        delivered := delivered ∪ {m};
        if (p, m) ∉ past then
            append(past, (p, m));
```

Garbage-Collection of Causal Past in the "No-Waiting Causal Broadcast"

Implements:
CausalOrderReliableBroadcast, **instance** *crb*.

Uses:
ReliableBroadcast, **instance** *rb*;
PerfectFailureDetector, **instance** $\mathcal{P}$.

```
// Except for its ⟨ Init ⟩ event handler, the pseudo code on the left is
// part of this algorithm.

upon event ⟨ crb, Init ⟩ do
    delivered := ∅;
    past := [];
    correct := Π;
    forall m do ack[m] := ∅;

upon event ⟨ P, Crash | p ⟩ do
    correct := correct \ {p};

upon exists m ∈ delivered such that self ∉ ack[m] do
    ack[m] := ack[m] ∪ {self};
    trigger ⟨ rb, Broadcast | [ACK, m] ⟩;

upon event ⟨ rb, Deliver | p, [ACK, m] ⟩ do
    ack[m] := ack[m] ∪ {p};

upon correct ⊆ ack[m] do
    forall (s', m') ∈ past such that m' = m do
        remove(past, (s', m));
```

Exercise 6 (Solution)

When removing a message m from the past, we can also remove all the messages that causally precede this message — and then recursively those that causally precede these.

Exercise 7

Can we devise a Best-effort Broadcast algorithm that satisfies the causal delivery property, *without* being a causal broadcast algorithm, i.e., without satisfying the *agreement* property of a reliable broadcast?

Exercise 7 (Solution 1/2)

No! Assume that some broadcast algorithm ensures the causal delivery property and is not reliable, but best-effort; define an instance *co* of the corresponding abstraction, where processes *co*-broadcast and *co*-deliver messages.

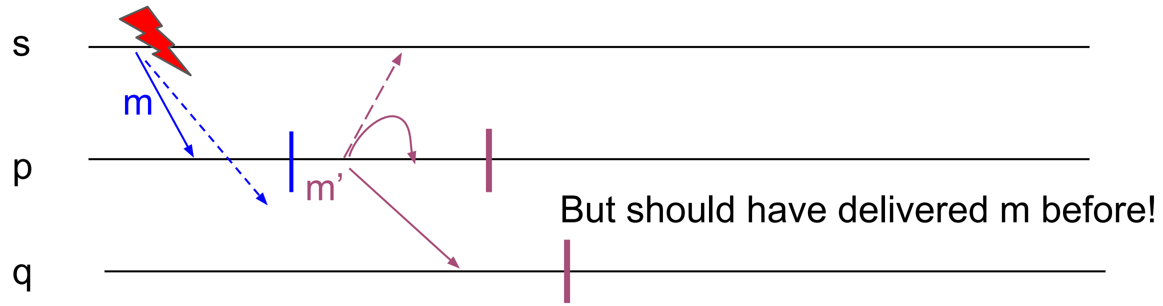
The only way for an algorithm to be best-effort broadcast but not reliable broadcast is to violate the agreement property: there must be some execution of the algorithm where some correct process p *co*-delivers a message m that some other process q does not ever *co*-deliver. This is possible in a best-effort broadcast algorithm, in fact this can only happen if the process s that *co*-broadcasts the message m is faulty (and crashes during the broadcast of m).

Exercise 7 (Solution 2/2)

Assume now that after *co*-delivering m, process p co-broadcasts a message m'. Given that p is correct and that the broadcast is best-effort, all correct processes, including q, will co-deliver m'. Given that m precedes m' (in causal order), q must have co-delivered m as well, a contradiction.

Hence, any best-effort broadcast that satisfies the causal delivery property satisfies agreement and is, thus, also a reliable broadcast.

In a nutshell:



Exercise 8

In the “Waiting Causal Broadcast”, we say that V ≤ W if, for every i = 1, …, N, it holds that V[i] ≤ W[i].

Question: Why do we not use “<” instead of “≤”?

Algorithm 3.15: Waiting Causal Broadcast

```
Implements:
    CausalOrderReliableBroadcast, instance crb.

Uses:
    ReliableBroadcast, instance rb.

upon event ⟨ crb, Init ⟩ do
    V := [0]^N;
    lsn := 0;
    pending := ∅;

upon event ⟨ crb, Broadcast | m ⟩ do
    W := V;
    W[rank(self)] := lsn;
    lsn := lsn + 1;
    trigger ⟨ rb, Broadcast | [DATA, W, m] ⟩;

upon event ⟨ rb, Deliver | p, [DATA, W, m] ⟩ do
    pending := pending ∪ {(p, W, m)};
    while exists (p′, W′, m′) ∈ pending such that W′ ≤ V do
        pending := pending \ {(p′, W′, m′)};
        V[rank(p′)] := V[rank(p′)] + 1;
        trigger ⟨ crb, Deliver | p′, m′ ⟩;
```

Exercise 8 (Solution)

Let V be encoding of the past of process q, and W be the encoding of the sender s *at the moment* of sending a message m.
“V[p] = W[p]” means that q is not “missing” any messages from p that s had delivered before it sent m. Hence, q should not wait for any other messages with sender p and should deliver m.

Example: Suppose that s broadcasts m with the vector clock [0, …, 0]. Then, no process delivers m if we use “<” instead of “≤”.