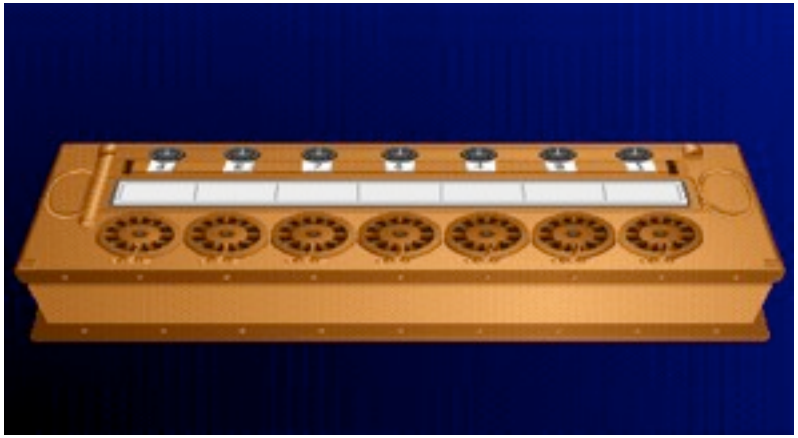
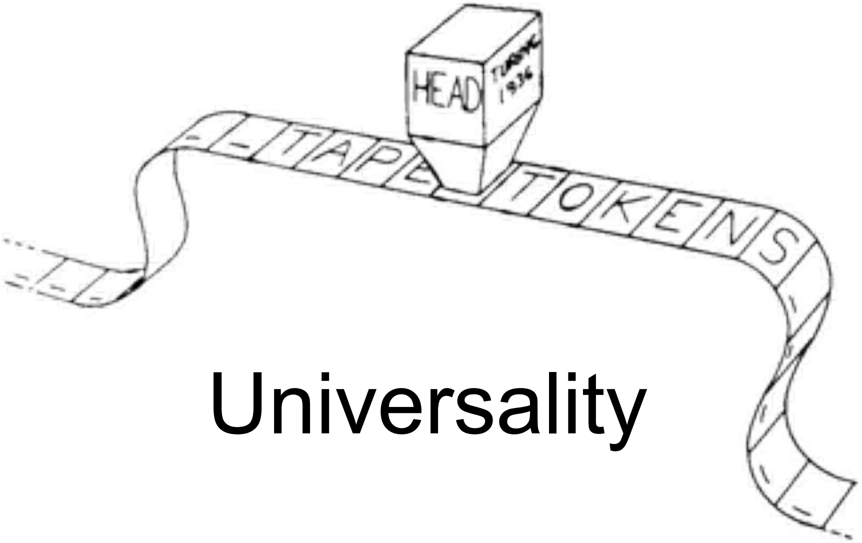
Generalized Universality

Once upon a time





Universality

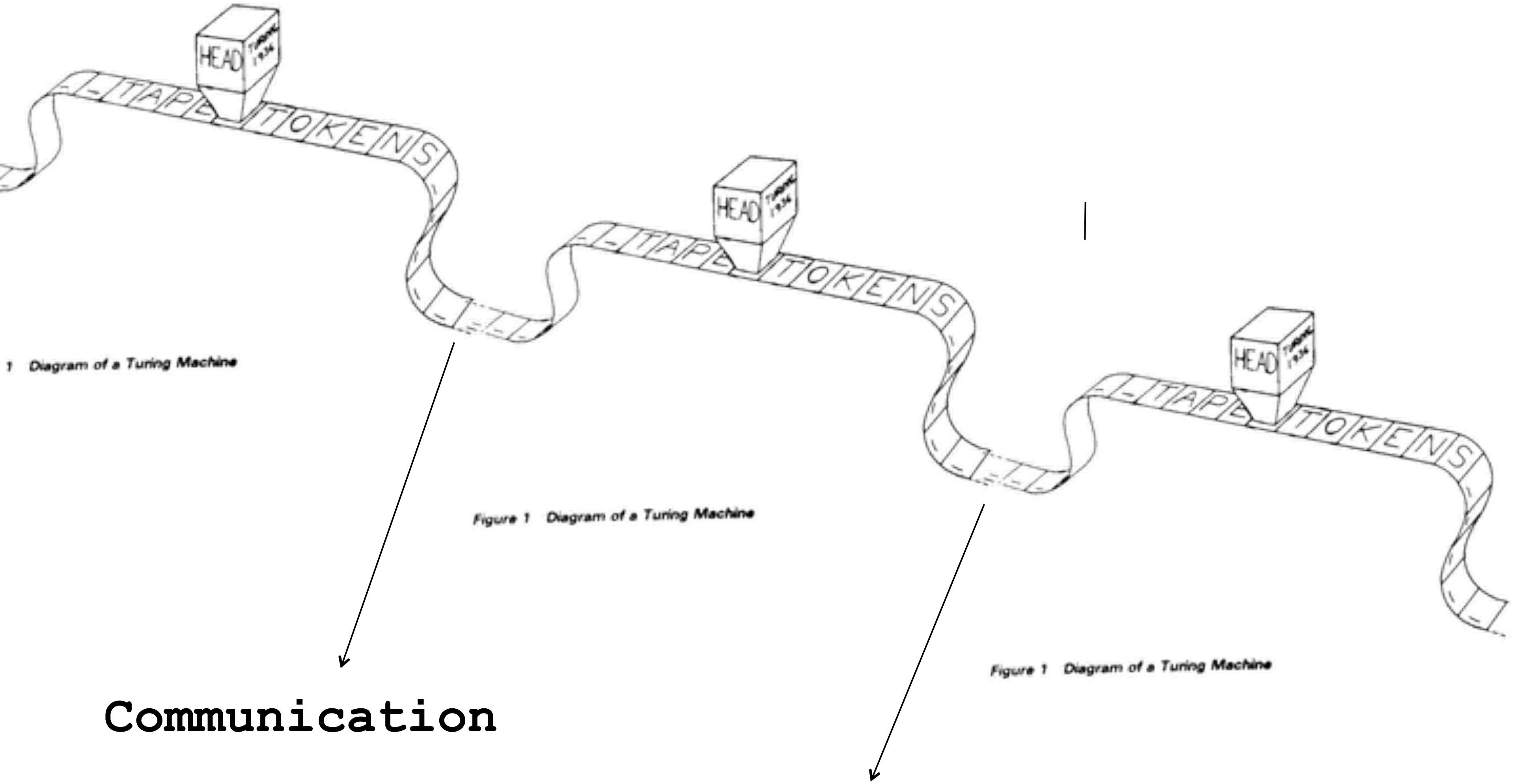
Algorithm

A finite set of instructions

The only intelligence required is to compute the instructions

Must always produce a result

Universality Today?


HEAD
TURING 1934
TAPE
TOKENS
1 Diagram of a Turing Machine
HEAD
TURING 1934
TAPE
TOKENS
Figure 1 Diagram of a Turing Machine
HEAD
TURING 1934
TAPE
TOKENS
Figure 1 Diagram of a Turing Machine

Communication

Communication

Algorithm

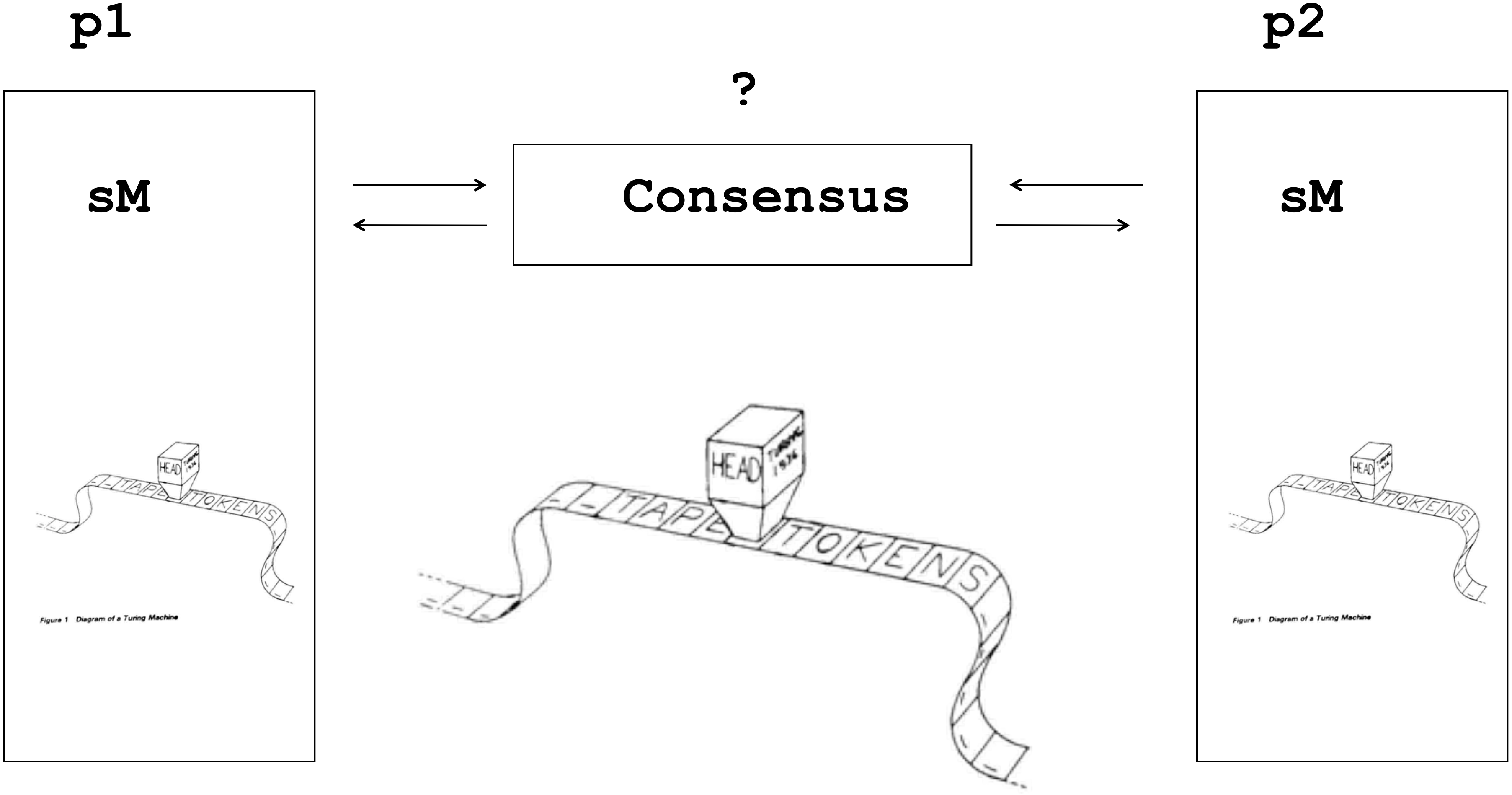
A finite set of precise instructions

The only intelligence required is to compute the instructions

Must **always** produce a result

NB. Despite concurrency and failures

Universality Today?



p1

p2

?

sM

Consensus

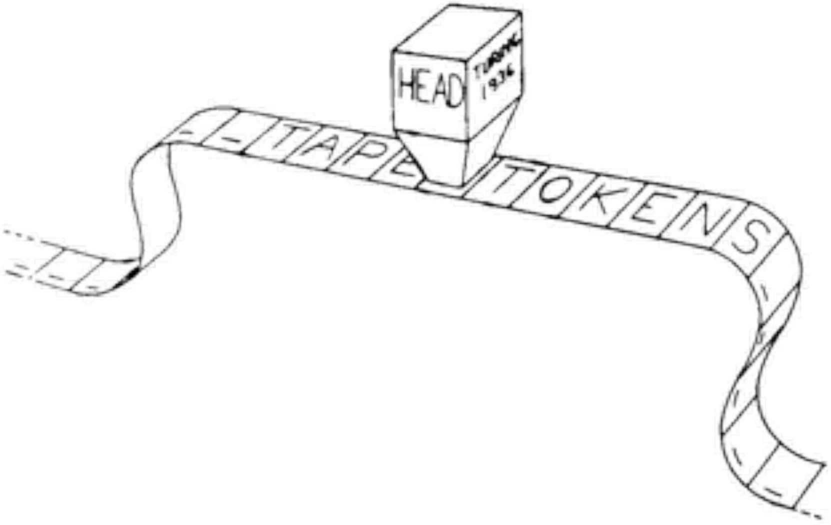
sM

Figure 1 Diagram of a Turing Machine

Figure 1 Diagram of a Turing Machine

Figure 1 Diagram of a Turing Machine

Universality of Consensus



Linearizable
(atomic)

Highly-available
(wait-free)

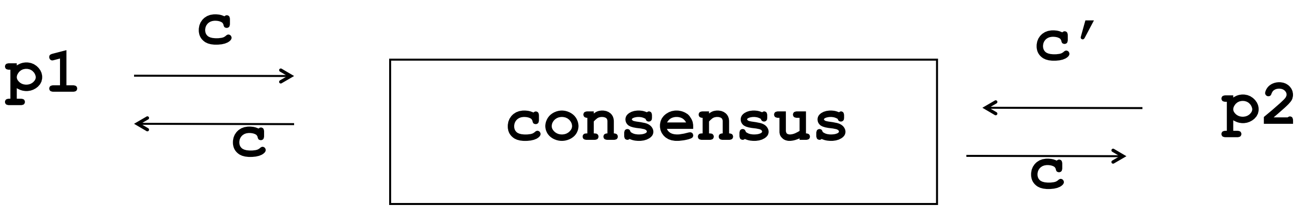
Act1: Universality

Act 2: Modern Universality

Consensus

Processes propose each a value and ***agree*** on one

output = ***propose(***input***)***



p1

c

c

consensus

c’

p2

c

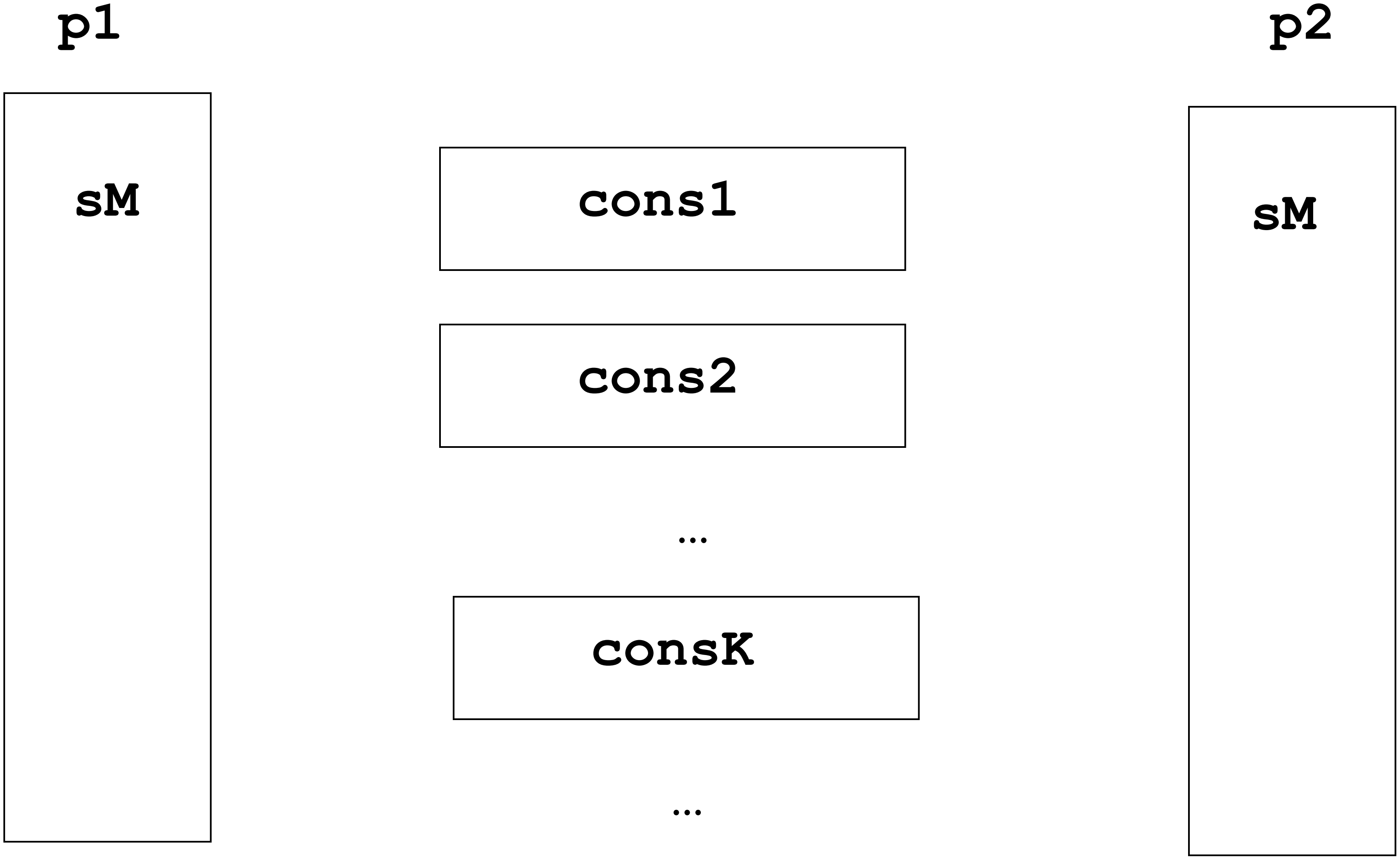
Universal Construction

Each process holds a copy of the - simulated - machine

Each process holds a list of commands for the machine

All processes share a list of consensus objects

Universal Construction



p1

sM

cons1

cons2

...

consK

...

p2

sM

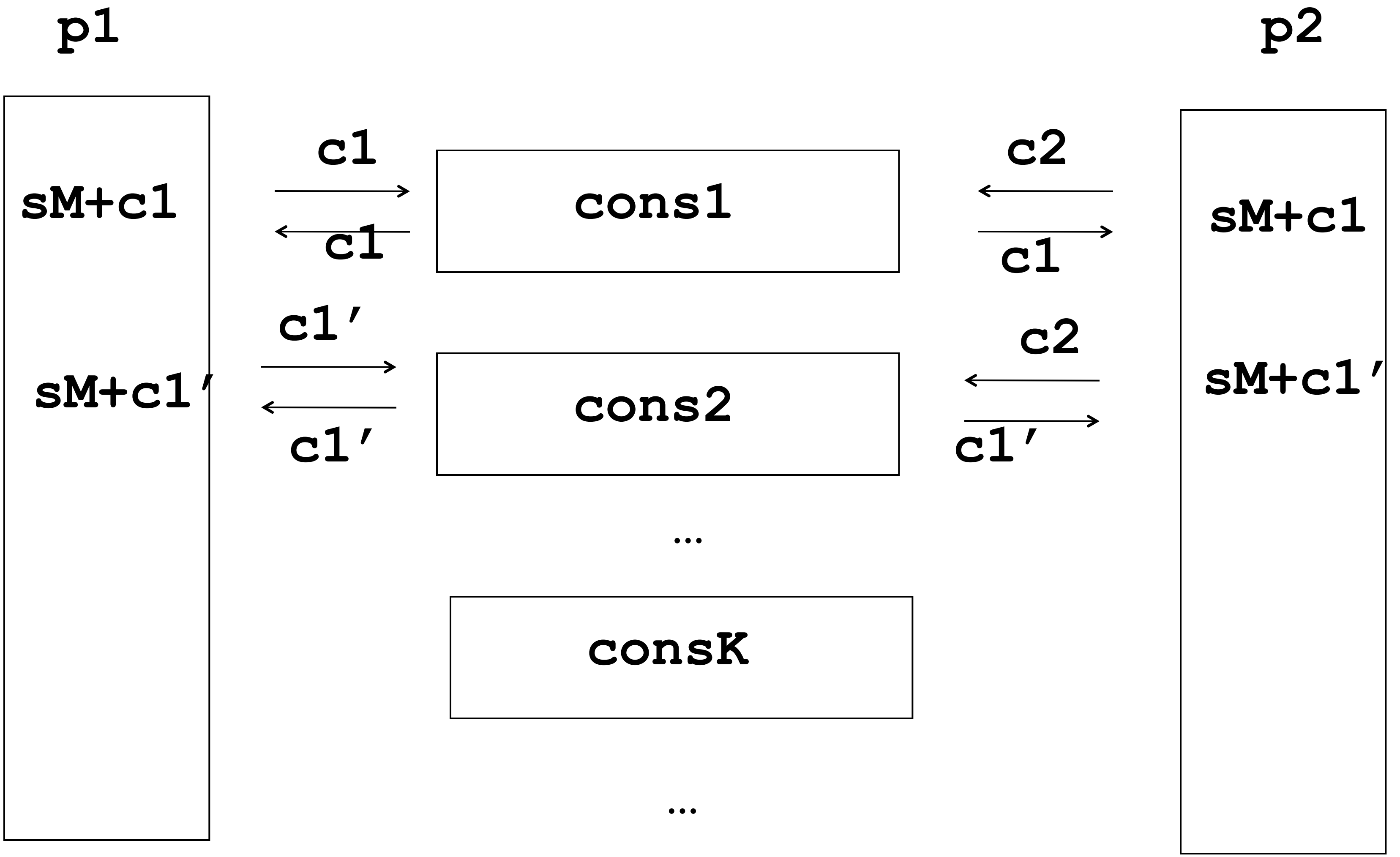
Universal Construction

- while(true)

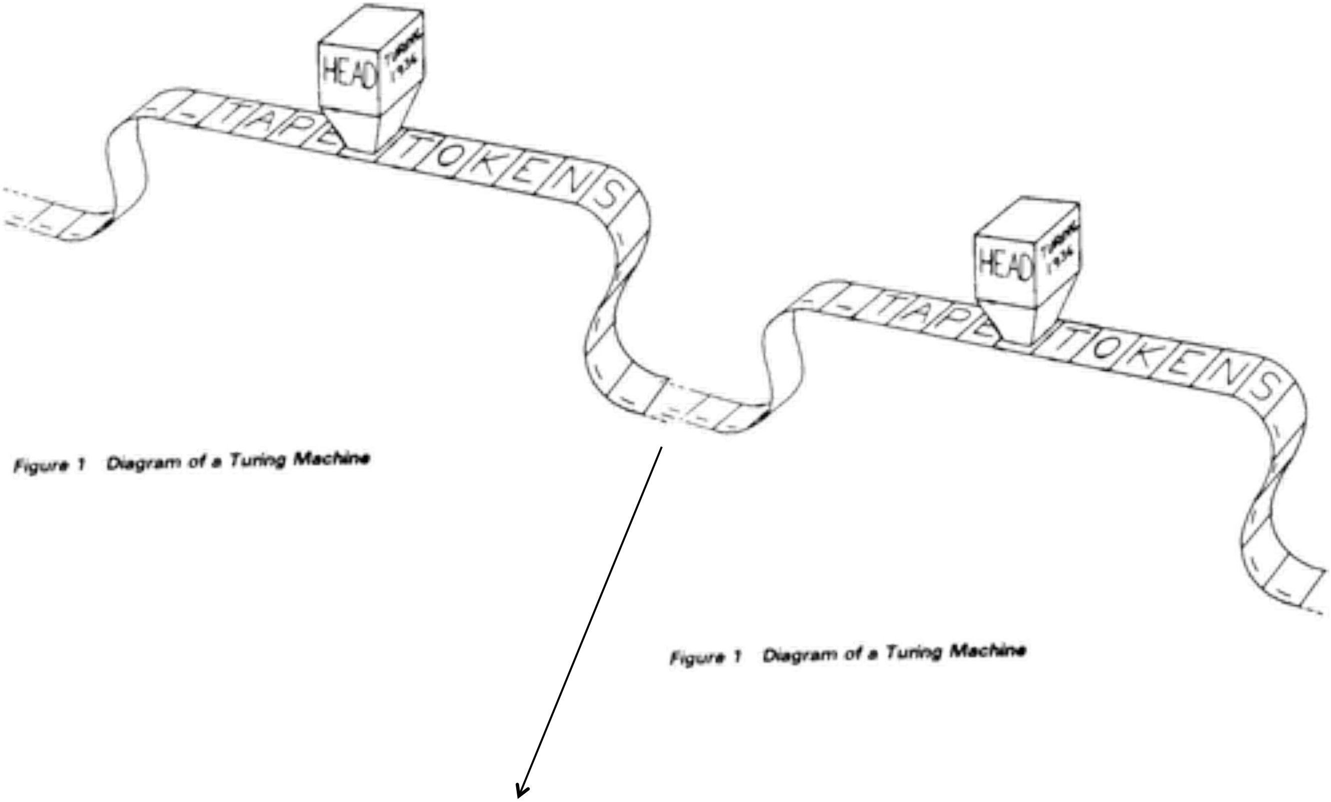
- c = commands.next()
- cons = Consensus.next()

- c’ = cons.***propose***(c)
- sM.***perform***(c’)

Universal Construction



What if consensus is not ensured?

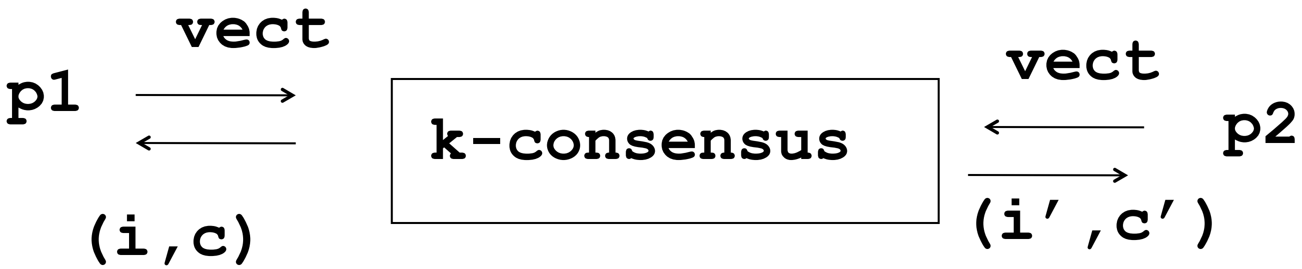

HEAD
TURING 1934
TAPE
TOKENS
Figure 1 Diagram of a Turing Machine
HEAD
TURING 1934
TAPE
TOKENS
Figure 1 Diagram of a Turing Machine

Consensus

K-Consensus

Every process proposes a vector of k values and returns a value at some position (Chauduri et al)

(i,c) = ***propose(***kVect***)***



vect

p1

(i,c)

k-consensus

vect

p2

(i',c')

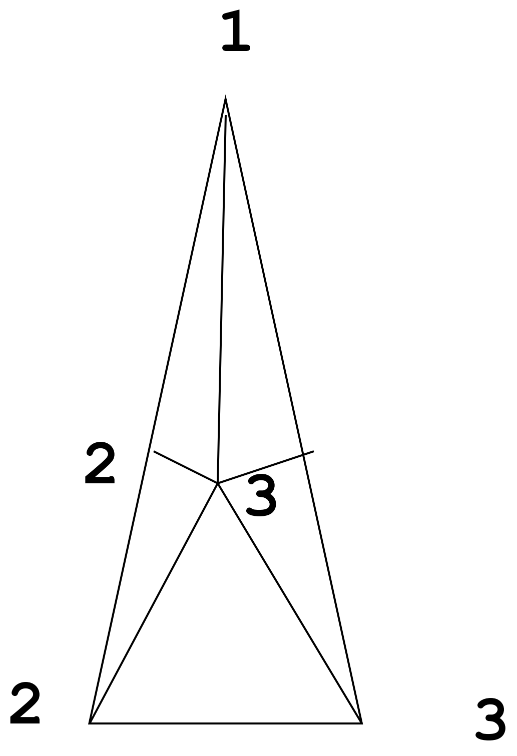
K-Consensus

- ***Validity***: the value returned at any position has been proposed at that position

- ***Agreement***: no two values returned at the same position are different

- ***Termination***: every correct process that proposes eventually returns

k+1-consensus is strictly weaker than k-consensus in any system of more than k+1 processes (Godel prize 2004 – HS,BG,SZ 93)



Sperner's Lemma: at least one triangle has three colors

What form of universality with K-consensus?

With consensus

We implement a highly-available state machine

With k-consensus

We implement k state machines of which ***at least one*** is highly-available

Generalized Universality

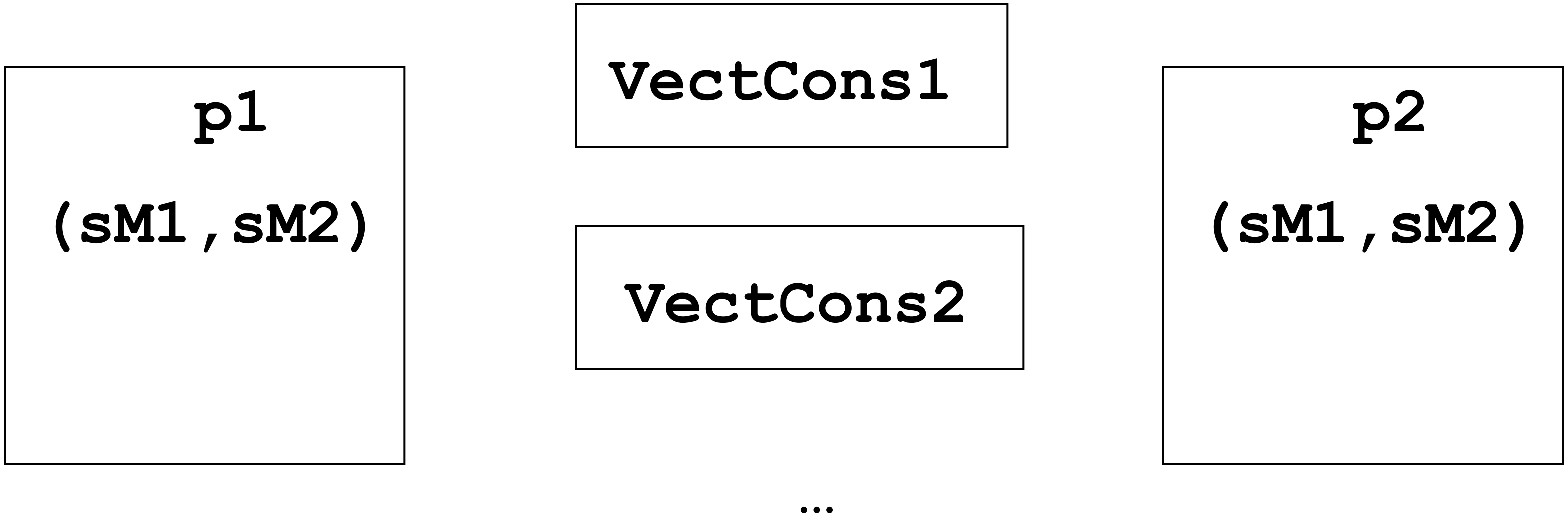
Act1: Universality

Act 2: Modern Universality

Act 3: Generalized Universality

Generalized Universality

Each process holds a copy of each of the machines sM(i) - and a lists of commands for each



p1

(sM1,sM2)

VectCons1

VectCons2

...

p2

(sM1,sM2)

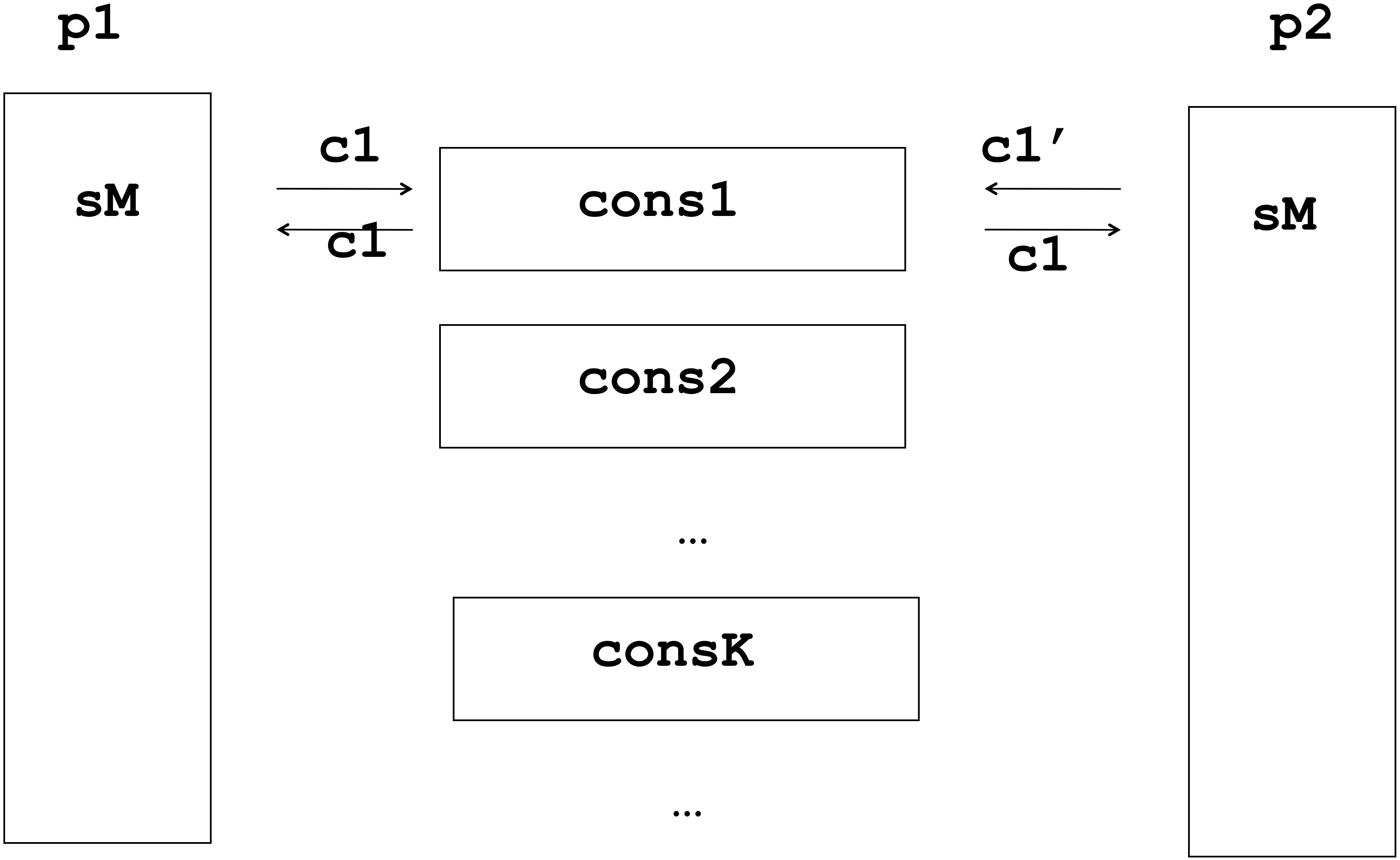
The processes share a list of k-vector consensus objects

Universal Construction

```
while(true)
    c = commands.next()
    cons = consensus.next()

    c’ = cons.propose(c)
    sM.perform(c’)
```

Universal Construction

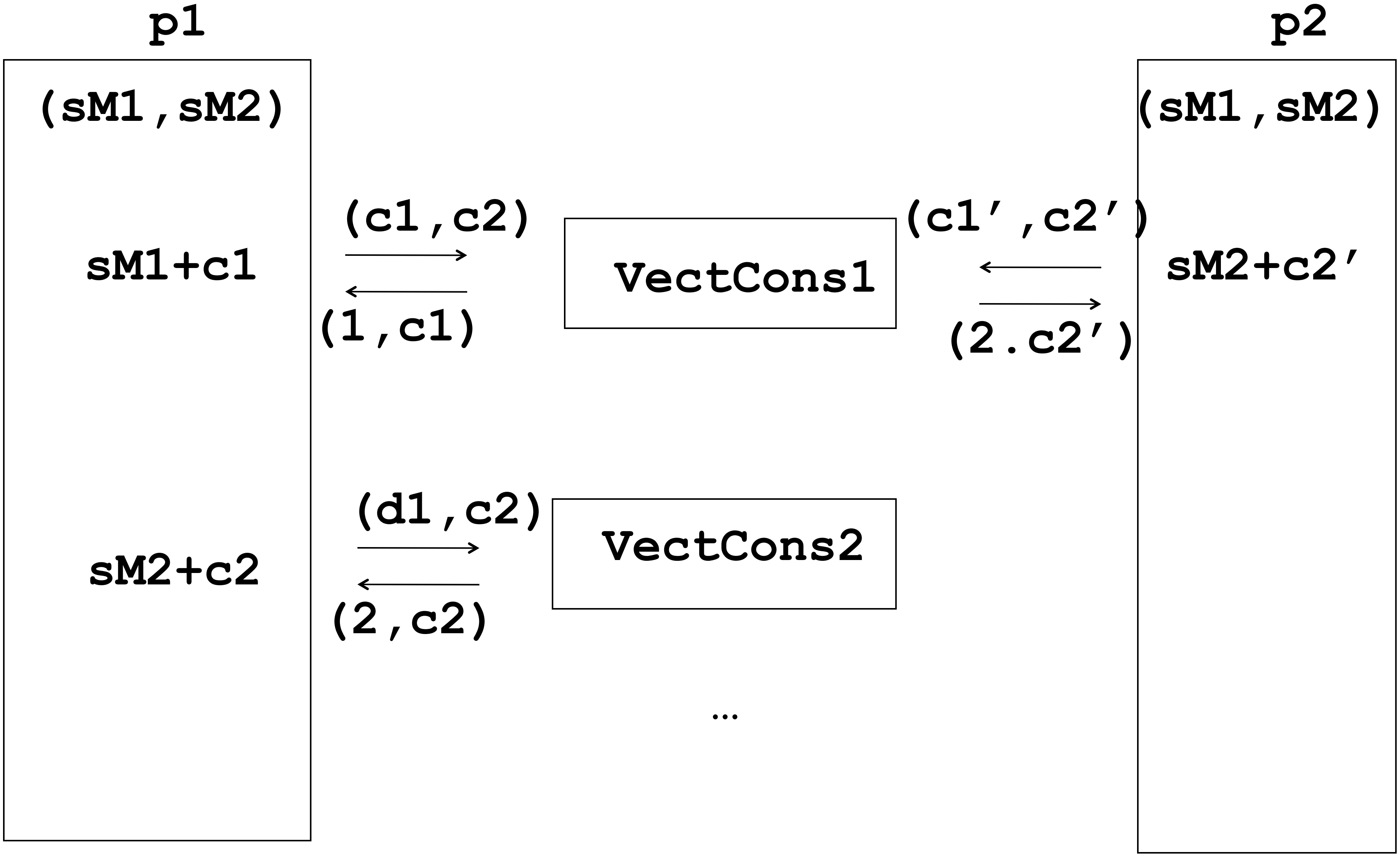


Generalized Universality?

```
while(true)
    for j = 1 to k: com(j) = commands(j).next()
    kVectC = kVectCons.next()

    (c,i) = kVectC.propose(com)
    sM(i).perform(c)
```

Problem with safety



p1

(sM1,sM2)

sM1+c1

(c1,c2)

(1,c1)

VectCons1

(c1',c2')

(2.c2')

p2

(sM1,sM2)

sM2+c2'

(d1,c2)

(2,c2)

VectCons2

sM2+c2

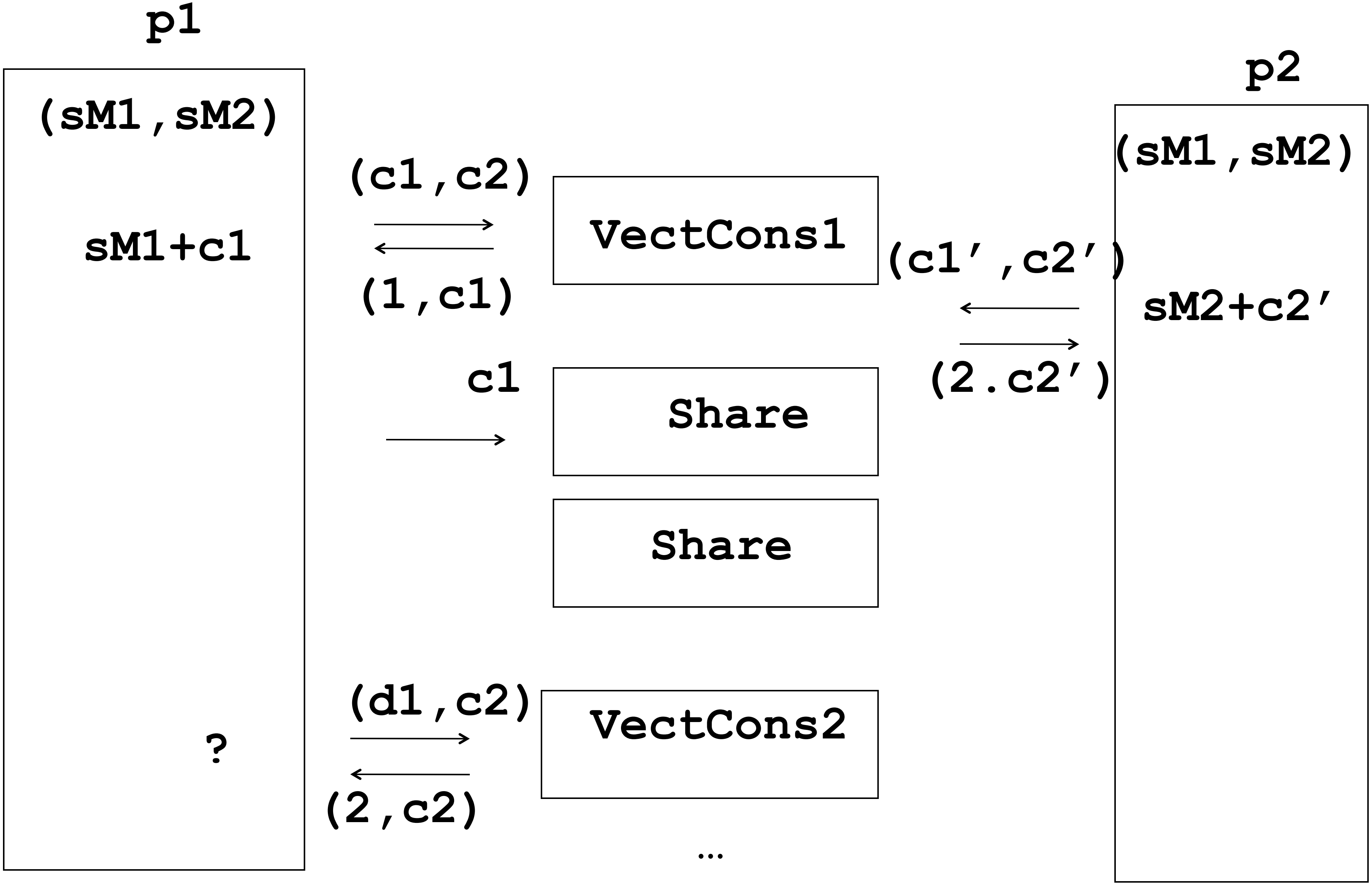
…

Generalized Universality

```
while(true)
  for j = 1 to k: com(j) = commands(j).next()
  kVectC = kVectCons.next()

  (c,i) = kVectC.propose(com)
  check other processes for any missing c’
  sM(i).perform(c)
  inform other processes about c
```

Generalized Universality



p1

(sM1,sM2)

sM1+c1

?

(c1,c2)

(1,c1)

VectCons1

c1

Share

Share

(d1,c2)

(2,c2)

VectCons2

…

p2

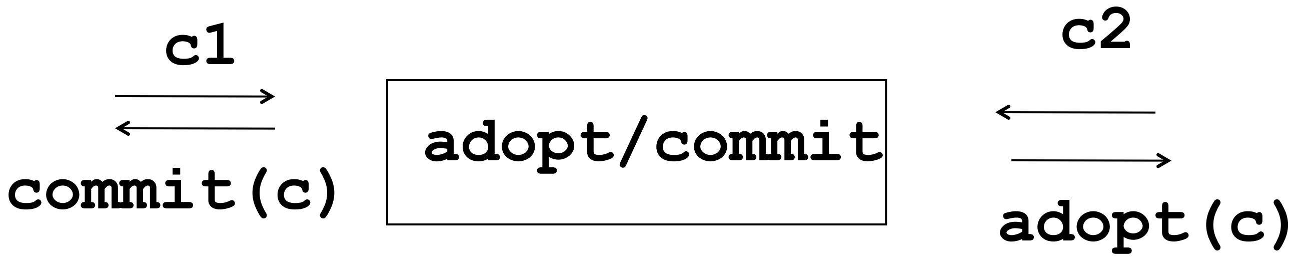
(sM1,sM2)

(c1′,c2′)

sM2+c2′

(2.c2′)

1st key idea (ensuring safety)



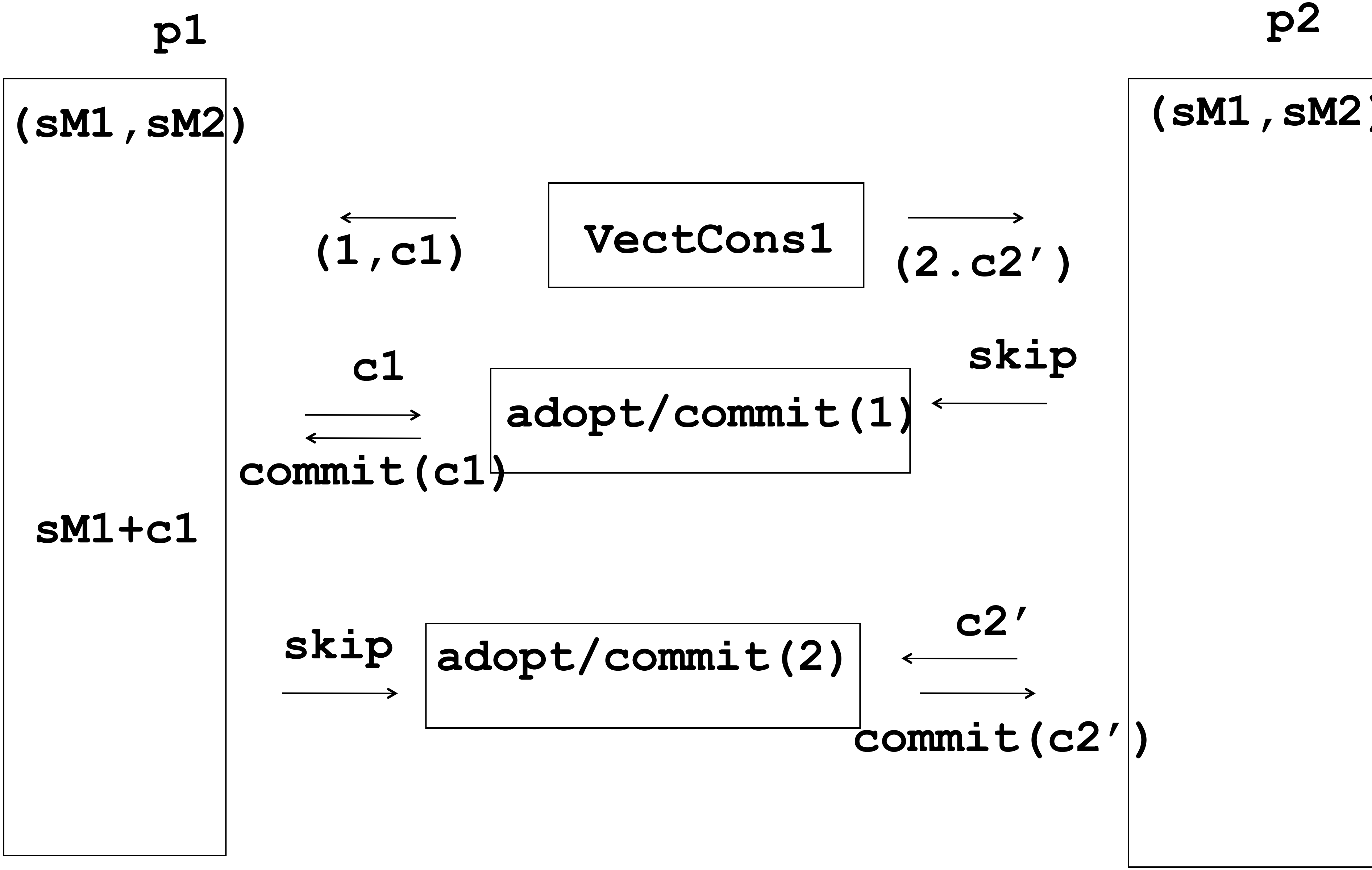
write (c)
if there is only c, write (commit, c)
if there is only (commit, c), return(commit, c)
if there is (commit, c’), return(adopt, c’)
else return (adopt, c)

Adopt/commit

- ***Invariant (1)***: if a value v is committed then no other value is returned

- ***Invariant (2)***: if all processes propose the same value then the value is committed

Generalized Universality



p1

p2

(sM1,sM2)

(sM1,sM2)

(1,c1)

VectCons1

(2.c2′)

c1

skip

adopt/commit(1)

commit(c1)

sM1+c1

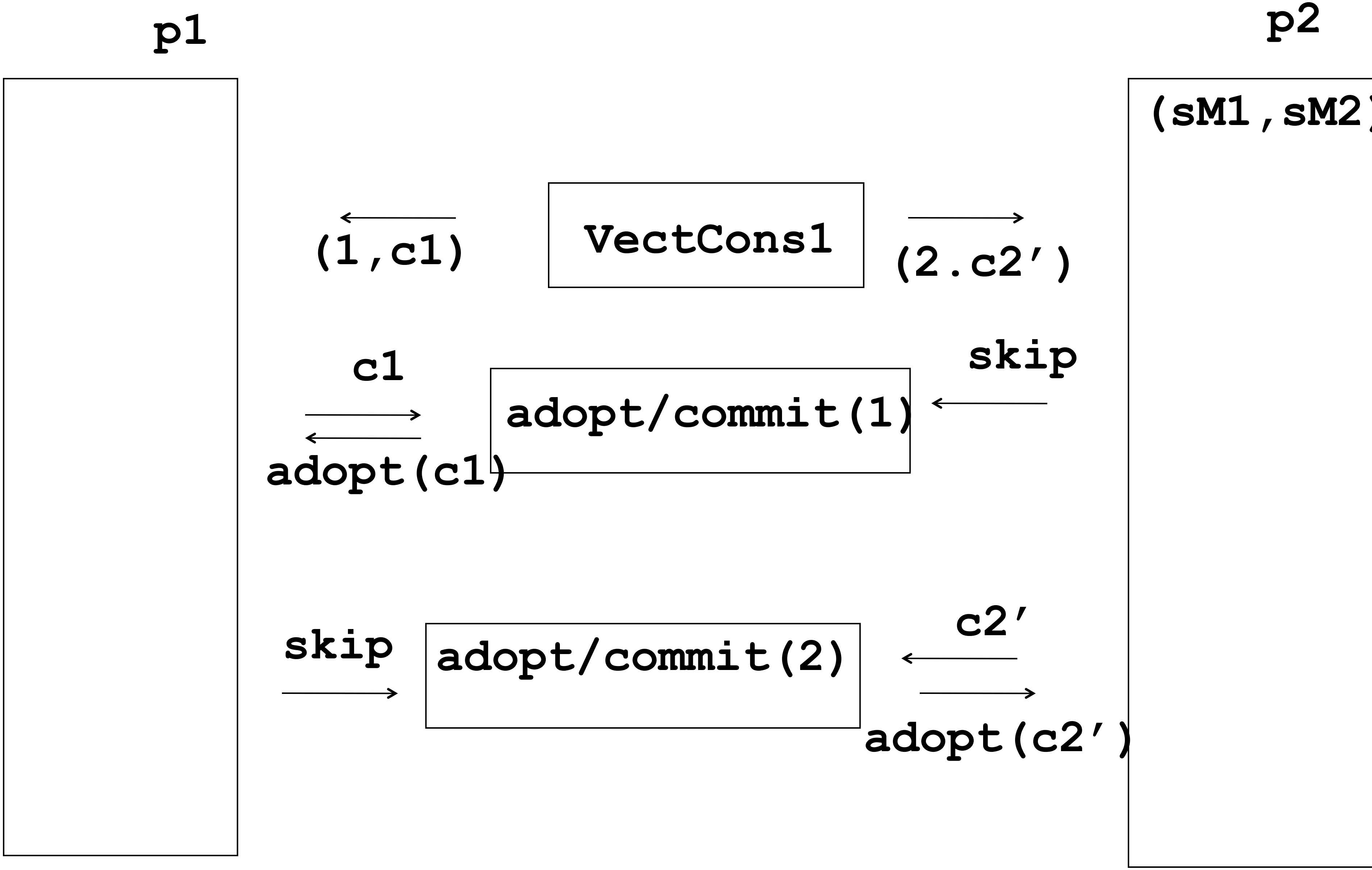
c2′

skip

adopt/commit(2)

commit(c2′)

Problem with liveness



p1

p2

(sM1,sM2)

VectCons1

(1,c1)

(2.c2′)

c1

adopt/commit(1)

skip

adopt(c1)

skip

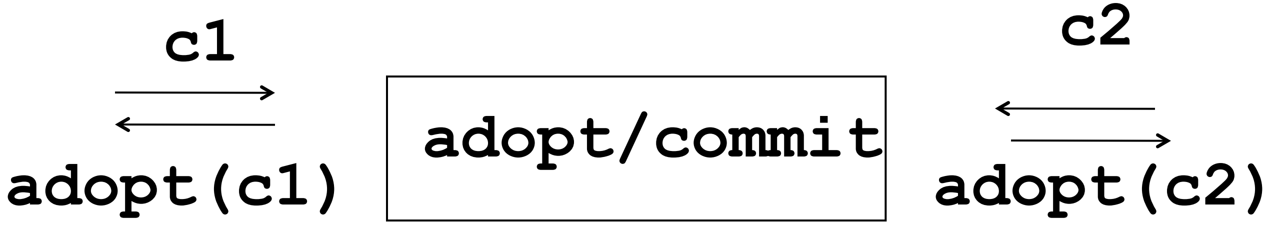
adopt/commit(2)

c2′

adopt(c2′)

2nd key idea (ensuring liveness)

Exploit success first



Can it be that no command is committed? i.e., if every adopt/commit box has one process proposes skip

Generalized universality (step 0)

- newCom = commands.next()
- while(true)
-   kVectC = kVectCons.next()

Generalized universality (step 1)

- …
- (c,i) = kVectC.propose(newCom)
- …

Generalized universality (step1-2)

- …
- (c,i) = kVectC.propose(newCom)
- vect(i) = commitment(i,c)
- …

Generalized universality (step1-2-2’)

- …
- (c,i) = kVectC.propose(newCom)
- vect(i) = commitment(i,c)
- for j = 1 to k except i:
    - vect(j) = commitment(newCom(j))

  …

Generalized universality (step 3)

```
…
for i = 1 to k
    if  ok(vect(i)) then
        sM(i).perform(vect(i))
        newCom(i) = commands(i).next()
    else
        newCom(i) = vect(i)
```

Generalized universality (step 3’)

```
…
for i = 1 to k
    If older(newCom(i),vect(i)) then
                                    sM(i).perform(newCom(i))
    If no(vect(i)) then newCom(i) = vect(i)
    else
    sM(i).perform(vect(i))
    If  vect(i) = newCom(i) then
        newCom(i) = commands(i).next()
    add(newCom(i),vect(i))
```

Commitment

- ***Safety***: a process does not perform a command unless all others know the command

- ***Liveness***: at least one process executes a command in every round

NB. Every correct process executes at least one command every two rounds

Act1: Universality

Act 2: Modern Universality

Act 3: Generalized Universality