The Limitations of Registers

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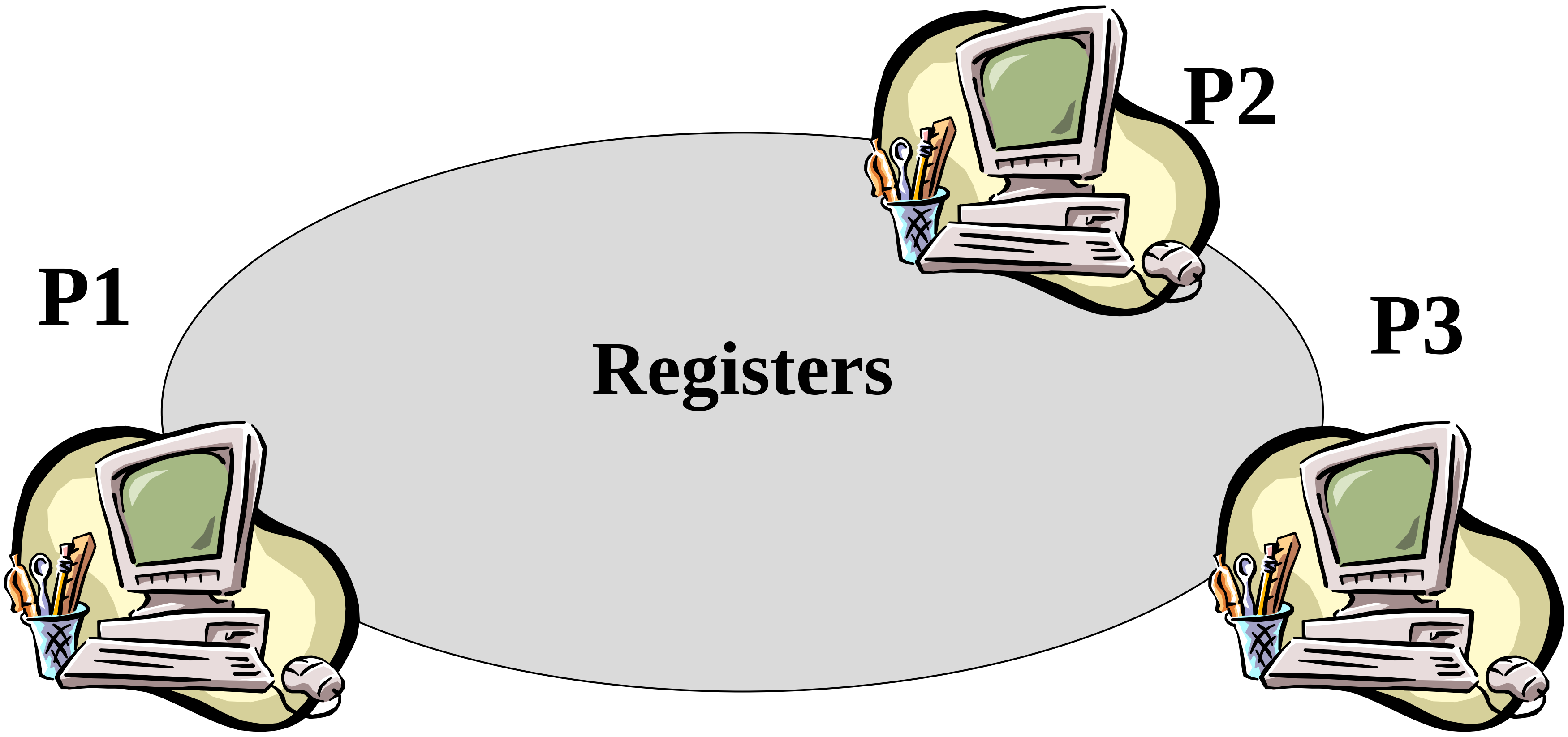
Registers

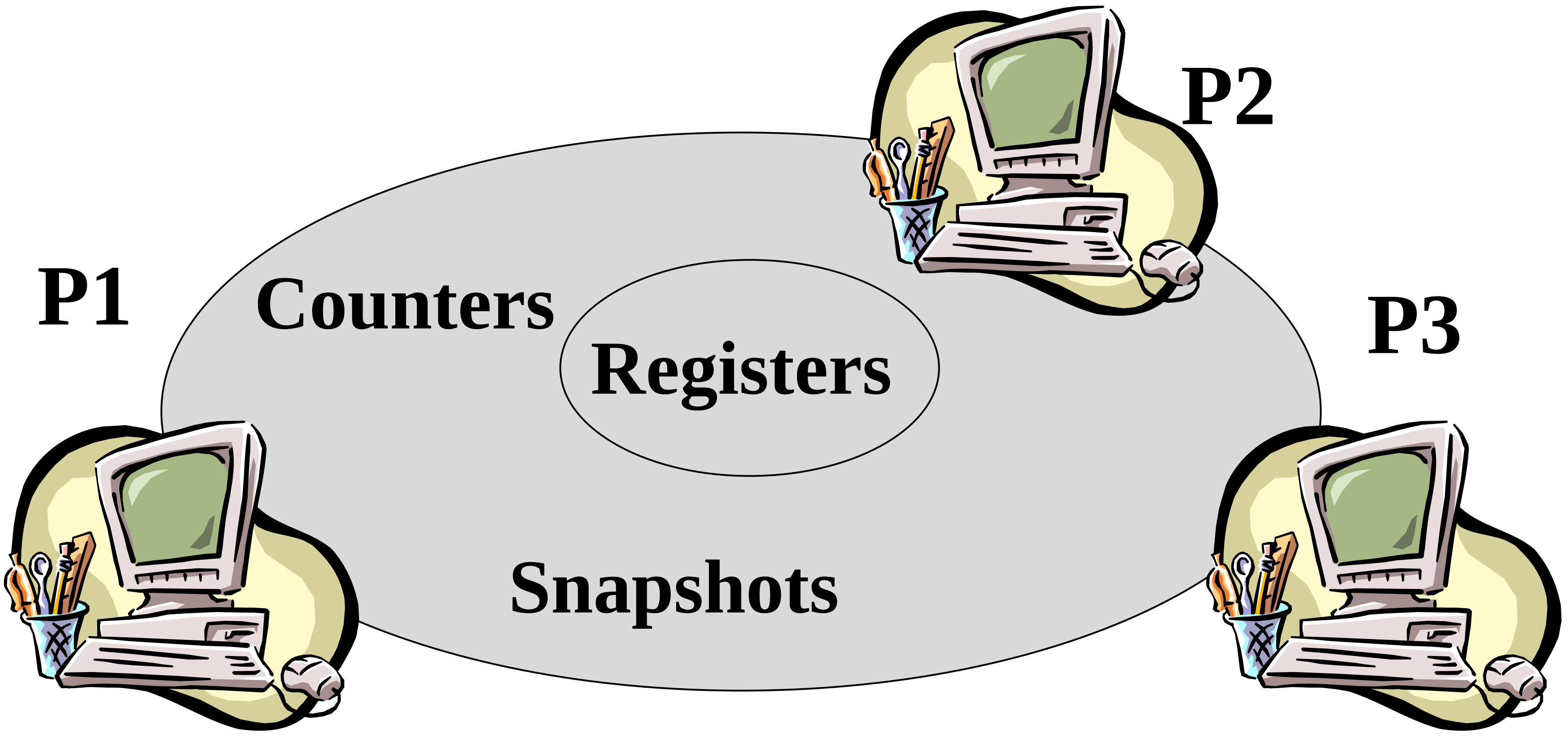
- ***Question 1:*** what objects can we implement with registers? ***Counters*** and ***snapshots*** (previous lecture)

- ***Question 2:*** what objects we cannot implement? (this lecture)

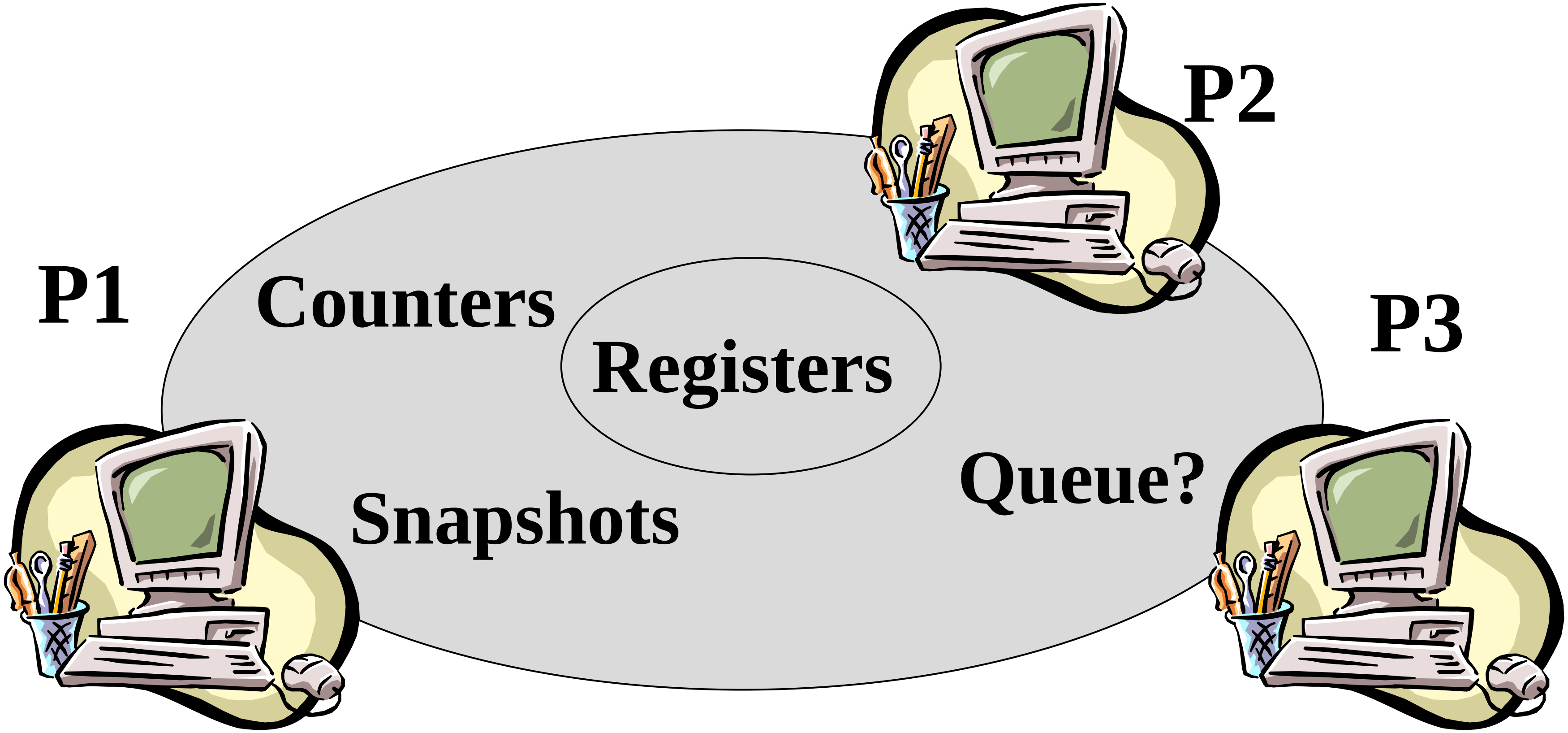
Shared memory model



Shared memory model



Shared memory model



Queue

- The queue is an object container with two operations: ***enq()*** and ***deq()***
- Can we implement a (atomic wait-free) ***queue***?

The consensus object

- One operation ***propose()*** which returns a value***.*** When a propose operation returns, we say that the process decides

- No two processes decide differently

- Every decided value is a proposed value

The consensus object

- ***Proposition:***
  - ✓ ***Consensus*** can be implemented among two processes with ***queues*** and ***registers***
- Proof (algorithm): consider two processes p0 and p1 and two ***registers*** R0 and R1 and a ***queue*** Q.

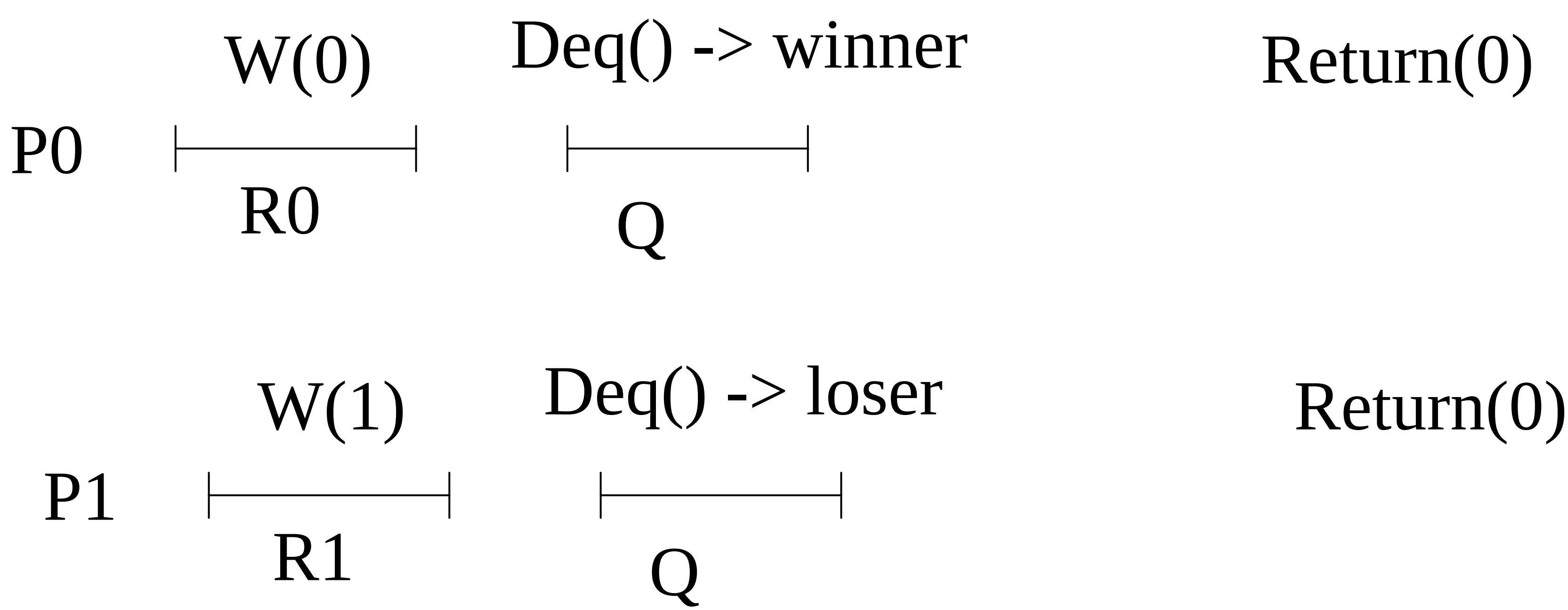
2-Consensus with queues

Uses two registers R0 and R1, and a queue Q

Q is initialized to {winner, loser}

Process pI:

```
propose(vI)
    RI.write(vI)
    item := Q.dequeue()
    if item = winner return(vI)
    return(R{1-I}.read())
```

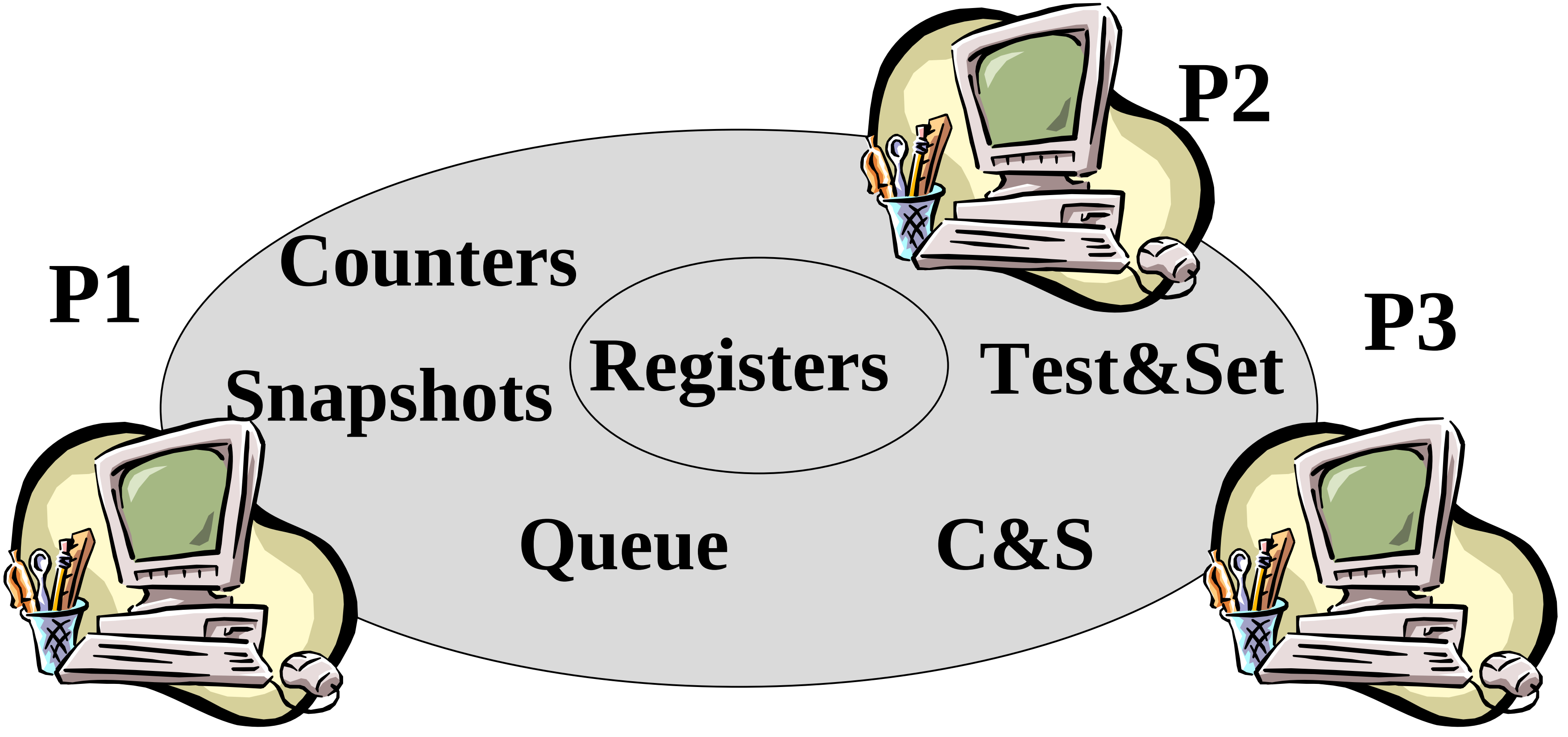

W(0)
Deq() -> winner
Return(0)
P0
R0
Q
W(1)
Deq() -> loser
Return(0)
P1
R1
Q

Correctness

Proof (algorithm):

- (wait-freedom) by the assumption of a wait-free register and a wait-free queue plus the fact that the algorithm does not contain any wait statement
- (validity) If pI dequeues winner, it decides on its own proposed value. If pI dequeues loser, then the other process pJ dequeued winner before. By the algorithm, pJ has previously written its input value in RJ. Thus, pI decides on pJ's proposed value;
- (agreement) if the two processes decide, they decide on the value written in the same register.

Shared memory model



2-Consensus with Counter

- Uses two registers R0 and R1, and a strong Counter object C (with one inc() operation that returns its value)
- (NB. The value in C is initialized to 0)

- Process pI:

- **propose(vI)**
- **RI.write(vI)**
- **val := C.inc()**
- **if(val = 1) then**
  - ✓ **return(vI)**
  - – **else return(R{1-I}.read())**

More consensus implementations

- A ***Test&Set*** object maintains binary values x, init to 0, and y; it provides one operation: ***test&set()***
  - ✓ Sequential spec:
  - ✓ test&set() {y := x; x: = 1; return(y);}

- A ***Compare&Swap*** object maintains a value x, init to ⊥, and provides one operation: ***compare&swap(v,w);***
  - ✓ Sequential spec:
    - c&s(old,new) {if x = old then x := new; return(x)}

2-Consensus with Test&Set

- Uses two registers R0 and R1, and a Test&Set object T
-
- Process pI:
- **propose(vI)**
- **RI.write(vI)**
- **val := T.test&set()**
- **if(val = 0) then**
  - ✓ **return(vI)**
  - **else return(R{1-I}.read())**

N-Consensus with C&S

- Uses a C&S object C (initialized to ⊥)
- Process pI:
- **propose(vI)**
- **val := C.c&s(⊥, vI)**
- **return(val)**

Impossibility [FLP85,LA87]

- ***Proposition:*** there is no algorithm that implements ***consensus*** among two processes using only ***registers***

- ***Corollary:*** there is no algorithm that implements a ***queue*** (***Scounter, Test&Set*** or ***C&S***) among two processes using only ***registers***

Registers

- ***Question 1:*** what objects can we implement with registers? ***Counters*** and ***snapshots*** (previous lecture)

- ***Question 2:*** what objects we cannot implement? All objects that (together with ***registers***) can implement ***consensus*** (this lecture)

Impossibility (Proof)

- ***Proposition:*** there is no algorithm that implements ***consensus*** among two processes using only ***registers***

- Proof (by contradiction): consider two processes p0 and p1 and any number of ***registers***, R1..Rk..

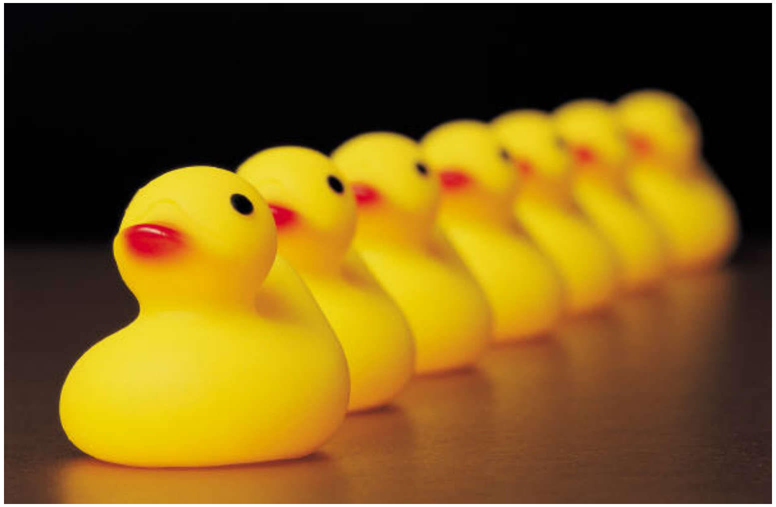
  Assume that a consensus algorithm A for p0 and p1 exists.

Elements of the model

- A ***configuration*** is a global state of the distributed system
- A new configuration is obtained by executing a ***step*** on a previous configuration: the step is the unit of execution

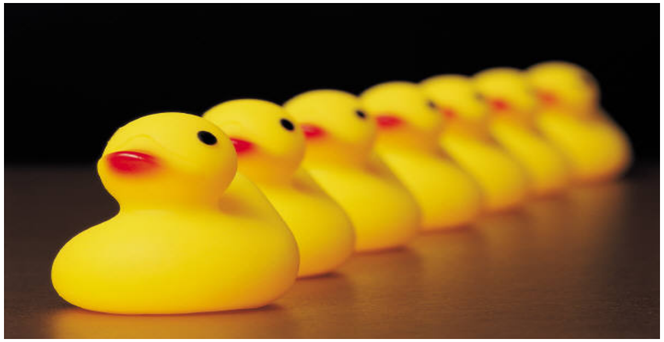
What is distributed computing?
A game

A game between an adversary and a set of processes



The adversary decides which process goes next

The processes take steps



Elements of the model

- The adversary decides which process executes the next step and the algorithm deterministically decides the next configuration based on the current one

Elements of the model

- ***Schedule:*** a sequence of steps represented by process ids
- The schedule is chosen by the system

- An asynchronous system is one with no constraint on the schedules: any sequence of process ids is a schedule

Consensus

- The algorithm must ensure that *agreement* and *validity* are satisfied in every schedule
- Every process that executes an infinite number of steps eventually decides

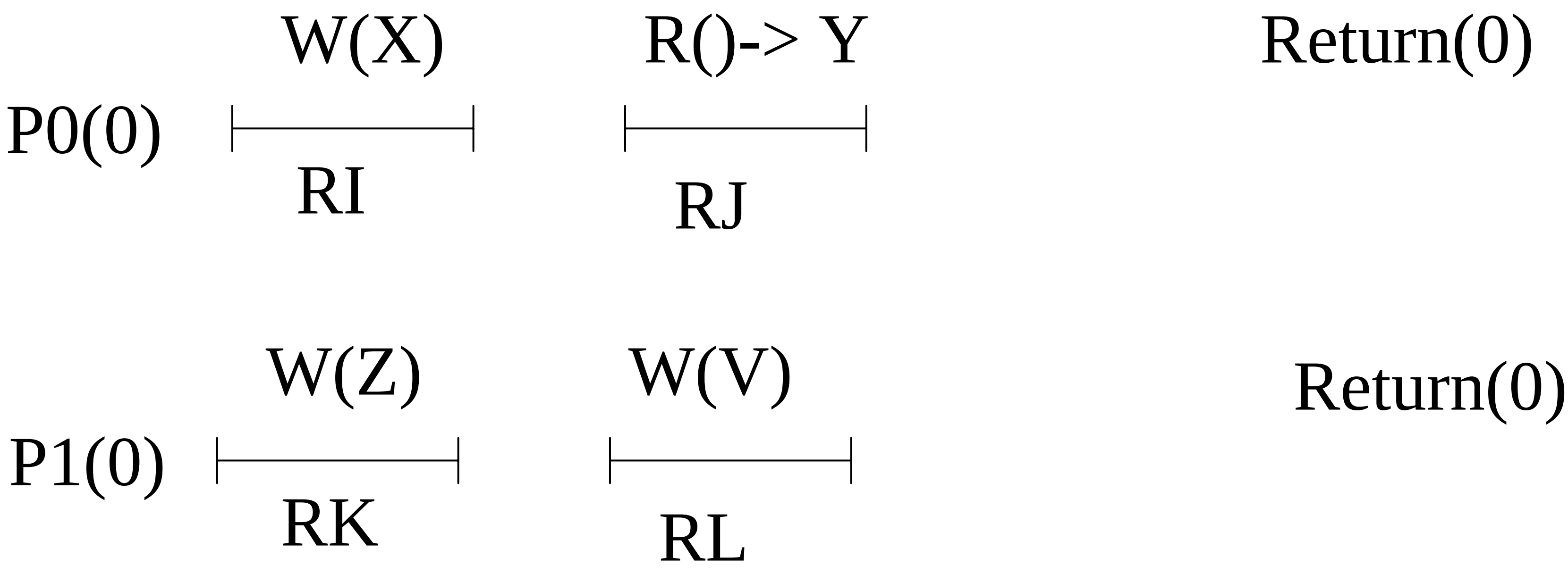
Impossibility (elements)

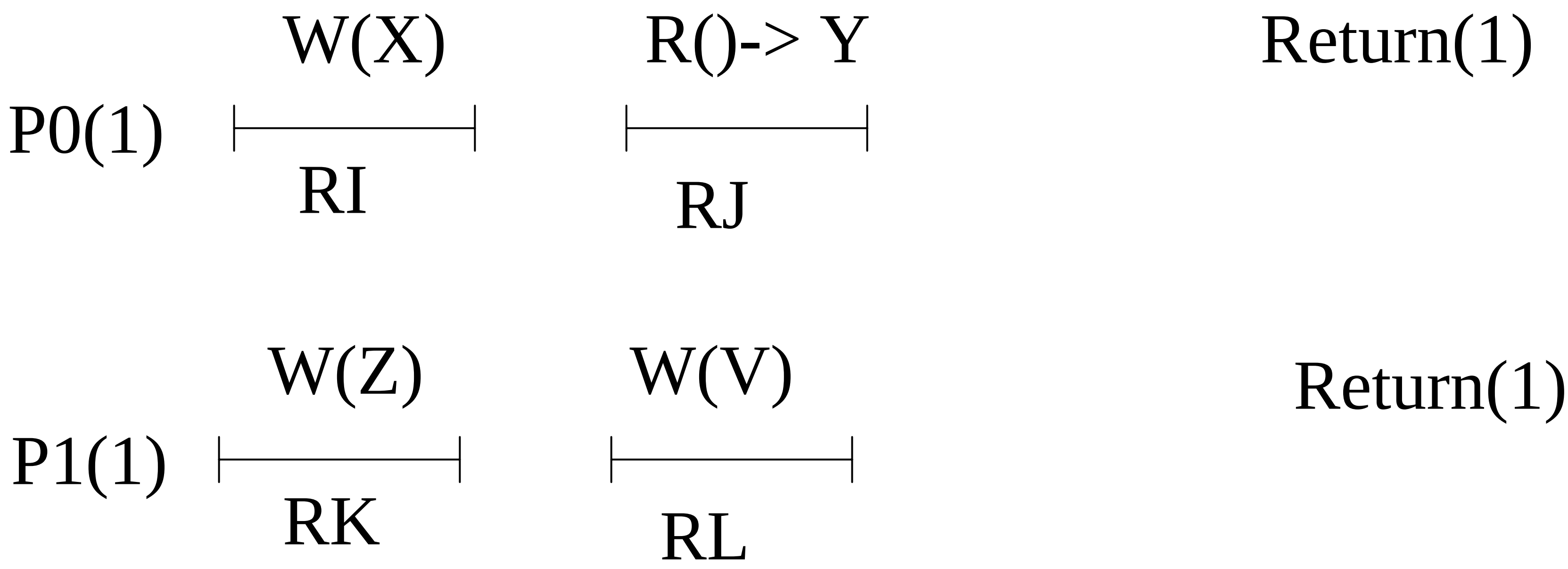
- (1) a (initial) ***configuration*** C is a set of (initial) values of p0 and p1 together with the values of the registers: R1..Rk,..;
- (2) a ***step*** is an elementary action executed by some process pI: it consists in reading or writing a value in a register and changing pI's state according to the algorithm A;
- (3) a ***schedule*** S is a sequence of steps; S(C) denotes the configuration that results from applying S to C.

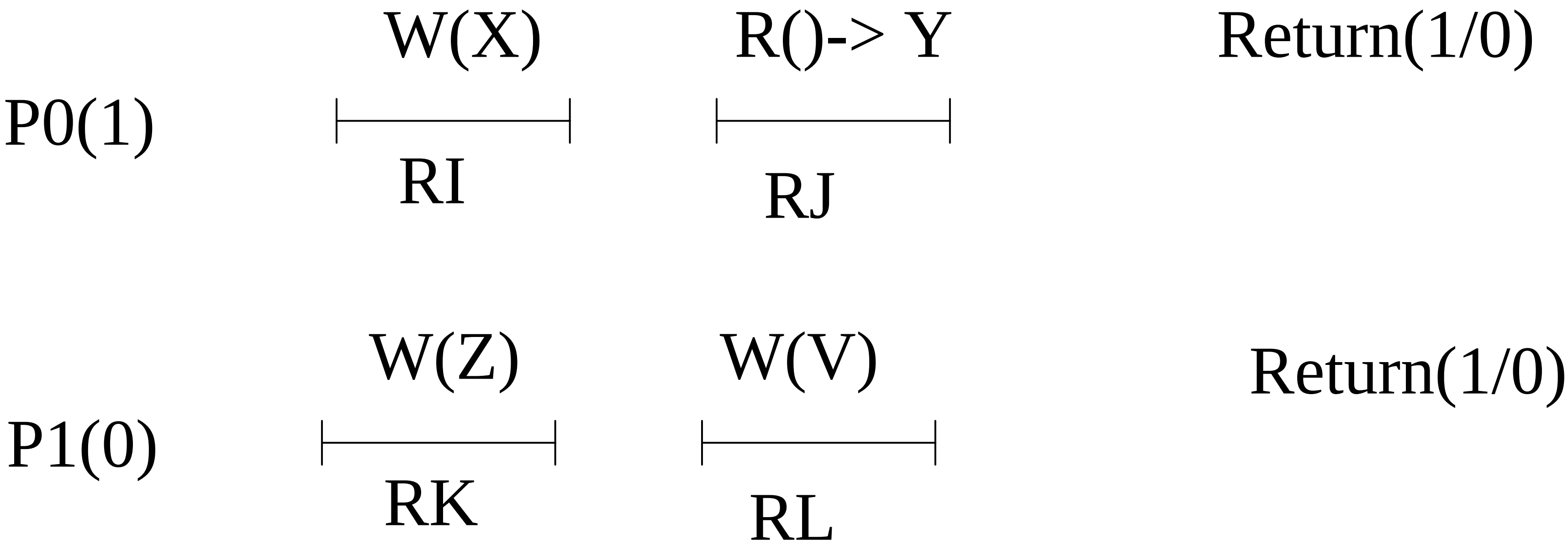
Impossibility (elements)

- Consider u to be 0 or 1; a configuration C is ***u-valent*** if, starting from C, no matter how the processes behave, no decision other than u is possible

- We say that the configuration is ***univalent***. Otherwise, the configuration is called ***bivalent***


W(X)
R()-> Y
Return(0)
P0(0)
RI
RJ
W(Z)
W(V)
Return(0)
P1(0)
RK
RL


W(X)
R()-> Y
Return(1)
P0(1)
RI
RJ
W(Z)
W(V)
Return(1)
P1(1)
RK
RL


W(X)
R()-> Y
Return(1/0)
P0(1)
RI
RJ
W(Z)
W(V)
Return(1/0)
P1(0)
RK
RL

Impossibility (structure)

- ***Lemma 1:*** there is at least one initial ***bivalent*** configuration

- ***Lemma 2:*** given any bivalent configuration C, there is an ***arbitrarily long schedule*** S(C) that leads to another bivalent configuration

The conclusion

- Lemmas 1 and 2 imply that there is a configuration C and an *infinite* schedule S such that, for any prefix S’ of S, S’(C) is bivalent.

- In infinite schedule S, at least one process executes an infinite number of steps and does not decide

- A contradiction with the assumption that A implements consensus.

Lemma 1

The initial configuration C(0,1) is bivalent

Proof: consider C(0,0) and p1 not taking any step: p0 decides 0; p0 cannot distinguish C(0,0) from C(0,1) and can hence decides 0 starting from C(0,1); similarly, if we consider C(1,1) and p0 not taking any step, p1 eventually decides 1; p1 cannot distinguish C(1,1) from C(0,1) and can hence decides 1 starting from C(0,1). Hence the bivalency.

Lemma 2

Given any bivalent configuration C, there is an arbitrarily long schedule S such that S(C) is bivalent

Proof (by contradiction): let S be the schedule with the maximal length such as D= S(C) is bivalent; p0(D) and p1(D) are both univalent: one of them is 0-valent (say p0(D)) and the other is 1-valent (say p1(D))

Lemma 2

- Proof (cont’d): To go from D to p0(D) (vs p1(D)) p0 (vs p1) accesses a register; the register must be the same in both cases; otherwise p1(p0(D)) is the same as p0(p1(D)): in contradiction with the very fact that p0(D) is 0-valent whereas p1(D) is 1-valent

Lemma 2

- Proof (cont'd): To go from D to p0(D), p0 cannot read R; otherwise R has the same state in D and in p0(D) ; in this case, the registers and p1 have the same state in p1(p0(D)) and p1(D); if p1 is the only one executing steps, then p1 eventually decides 1 in both cases: a contradiction with the fact that p0(D) is 0-valent; the same argument applies to show that p1 cannot read R to go from D to p1(D)

  Thus both p0 and p1 write in R to go from D to p0(D) (resp., p1(D)). But then p0(p1(D))= p0(D) (resp. p1(p0(D))= p1(D)) --- a contradiction.

The conclusion (bis)

Lemmas 1 and 2 imply that there is a configuration C and an *infinite* schedule S such that, for any prefix S' of S, S'(C) is bivalent.

In infinite schedule S, at least one process executes an infinite number of steps and does not decide

A contradiction with the assumption that A implements consensus.